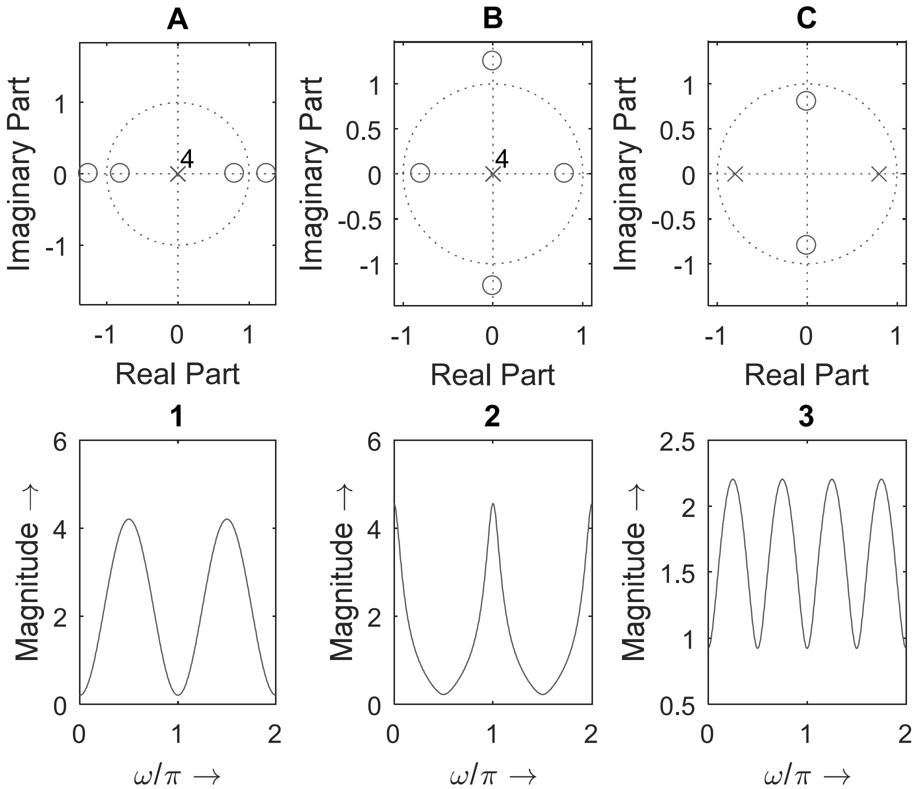
U.PORTO
FEUP FACULDADE DE ENGENHARIA UNIVERSIDADE DO PORTO

L.EEC/M.EEC
L.EEC025 - Fundamentals of Signal Processing

FIRST EXAM, JANUARY 17, 2023
Duration: 120 Minutes, closed book

NOTE: each question ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage.

1. Three causal discrete-time systems have the illustrated zero-pole diagrams A, B, and C, and the illustrated frequency response magnitudes 1, 2, and 3. Admit that the radius of all poles and zeros is either 0.8 or 1/0.8.



a) **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

b) **[1 *pt*]** Is any of the above systems (A, B, C) a linear-phase system ? Consider new systems formed by the following cascades: AB, AC, BC and ABC. Which of these new systems have a linear phase response ?

c) **[1 *pt*]** Comment on the following statement «Except for a constant gain, all systems verify $H(z) = H(-z)$». Is it true ? Is it false ? Why ?

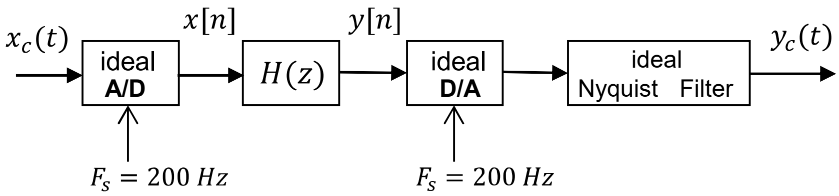
2. Consider that system C whose zero-pole diagram is represented in Prob. **1** is causal. The radius of all poles and zeros is $r = 0.8$.

a) **[1,5 *pts*]** Find the transfer function of the system, $H(z)$, write a difference equation implementing it and sketch a corresponding canonic realization structure.

b) **[1,5 *pt*]** Obtain a compact expression characterizing the magnitude of the frequency response of the system, $\left|H\left(e^{j\omega}\right)\right|$, and show that its maximum gain depends on $\left|\frac{1+r^2}{1-r^2}\right|$, and its minimum gain depends on $\left|\frac{1-r^2}{1+r^2}\right|$.

Note: You may assume here that $H(z) = \frac{1+\left(rz^{-1}\right)^2}{1-(rz^{-1})^2}$.

c) **[1 *pt*]** Consider the illustrated analog and causal discrete-time system whose transfer function is as suggested in **b)**. The sampling frequency is 200 Hz. The analog input signal is $x_c(t) = 1 + \sin(300\pi t) + 2\sin(500\pi t)$. Notice that an *anti-aliasing* filter does not exist.



Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e. in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

d) **[1 *pt*]** Presuming ideal reconstruction conditions, indicate what sinusoidal frequencies (in Hertz) exist in $y_c(t)$, and indicate what their magnitudes are.

Note: In case you did not solve **c)**, admit that the $x[n]$ sinusoidal frequencies are $\omega_0 = 0$ rad., $\omega_1 = \frac{\pi}{2}$ rad.

3. In one of the FPS Labs, the following code was used to service an interrupt-based routine whose most relevant C code is as follows (assume that N and `BETA` and `ALFA` are real-valued constants defined outside the scope of this routine, and `w[]` represents a vector of `N+1` floating point numbers initialized to zero outside the scope of this routine):

```
...
enum filtertype{FIR,IIR};
...
w0  = (float32_t)(rx_sample_L);

switch (myfilter)
{
     case FIR:
               yn = w0 + (float32_t)(ALFA) * w[N];
               break;
     case IIR:
               w0 = w0 + (float32_t)(BETA) * w[N];
               yn = w0;
              break;
     default:
              yn = w0;
}
w[0] = w0;
for (i=N ; i>0 ; i--) w[i] = w[i-1];

tx_sample_L = (int16_t)(yn);
...
```

a) **[1,5 *pts*]** Explain with words the operation of this code and write the difference equations it implements according to the type of selected filter (`myfilter`).

b) **[1,5 *pts*]** Now, admit that the code is modified to:

```
case IIR:
        w0 = w0 + (float32_t)(ALFA) * w[N];
        yn = w[N] - (float32_t)(ALFA) * w0;
   break;
```

Sketch the realization structure of the discrete-time system when `myfilter=IIR`. What does the new discrete-time system correspond to ? Justify.

4. Consider that $x[n]$ is an *N*-periodic signal and its DFT is $X[k]$.

a) **[2 *pts*]** Consider a 2*N*-periodic signal $Y[k]$ obtained as $Y[2k] = 2X[k]$, and $Y[2k+1] = 0$, for $k = 0,1,\dots,N-1$.
Show that $y[n] = x[(n)_N]$, $n = 0,1,\dots,2N-1$.

Now, consider the following Matlab code.

```
x=[1 2 3 4]; X=fft(x); L=2*length(x);
Y=zeros(1,L); Y(1:2:end)=2*X ;
y=ifft(Y)
 shift=L/4 ; Z=zeros(1,L);
Z(1+shift:L)=Y(1:L-shift); Z(1:shift)=Y(L-shift+1:L);
z=ifft(Z)
```

b) **[1 *pt*]** Without executing the code, find and explain the contents of vector `y`.

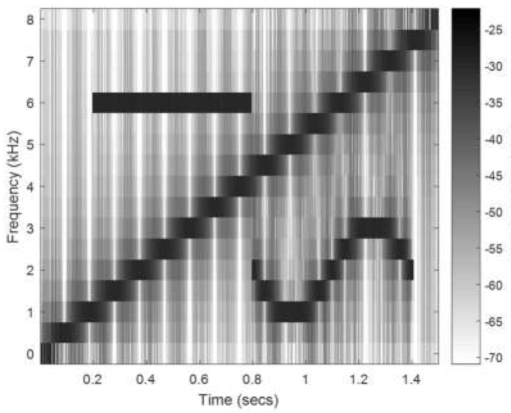
c) **[1,5 *pts*]** Without executing the code, find and explain the contents of vector `z`.

5. The sampling frequency of a real-valued audio signal is 16000 Hz and its spectral contents was analyzed using a sliding FFT with 50% overlap between adjacent FFTs. Two alternative FFT sizes were used: N (a number you need to identify) and 256, and two alternative windows were used: Rectangular and Hanning. The four resulting spectrograms are represented next in diagrams A, B, C and D.

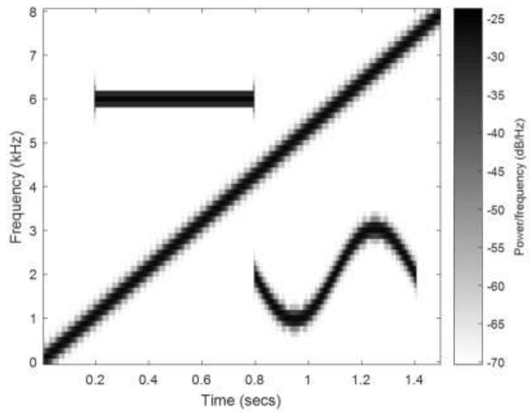
Note 1: Darker colors mean higher Power Spectral Densities

Note 2: the blurred effects in the spectrograms reflect the impact of signal processing and not printer problems

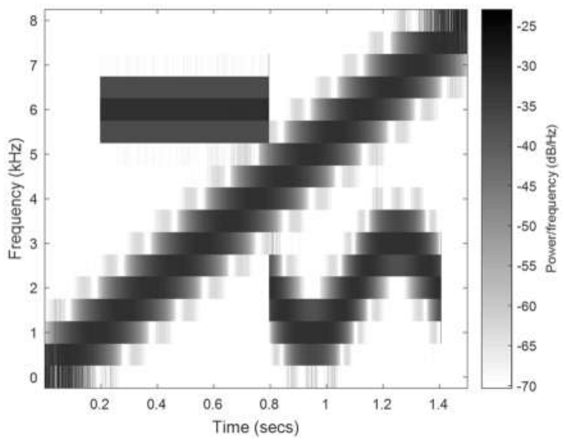
A



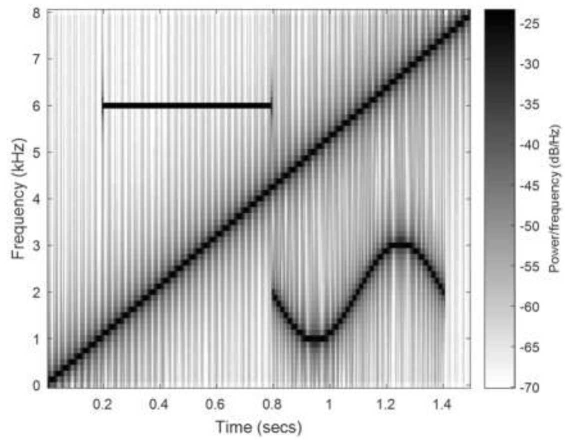
B



C



D



a) **[1,*5 pts*]** Represent schematically the sliding FFT analysis process and explain how it leads to the spectrogram representation.

b) **[1,*5 pts*]** Explain which diagrams (A, B, C, D) reflect the use of the Rectangular window, and which diagrams reflect the use of the Hanning window. Admitting that N is a power-of-two number, conclude on what the value of N is based on the observation of the diagrams.

c) **[1 *pts*]** Based on the observation of the diagrams, describe the spectral contents of the signal.

END