MESTRADO EM ENGENHARIA FÍSICA
EEC4002 - Processamento Estatístico de Sinal

Two-part closed book exam, except for the provided formulae sheet. Total duration: 120 minutes (parts 1 and 2)

1. **[3 pts]** The transfer function of a causal first-order discrete-time system is $H(z) = \frac{1}{1-\frac{2}{5}z^{-1}}$. The system is excited with zero-mean white noise whose variance is the unity. Find the auto-correlation sequence of the system output, $r_y[\ell]$.

   Hint: use frequency-domain analysis

2. **[3 pts]** Consider that $H(e^{j\omega})$ represents the frequency response of an ideal high-pass filter whose cut-off frequency is $\omega = \frac{3\pi}{4}$, that is, $H(e^{j\omega}) = 1$ for $\frac{3\pi}{4} < |\omega| < \pi$, and $H(e^{j\omega}) = 0$ elsewhere. If the filter is excited with zero-mean white noise having auto-correlation $r_x[\ell] = \sigma_x^2\delta[\ell]$, find the auto-correlation of the filter output, $r_y[\ell]$, as well as the average power of the noise at the filter output. Could this average power be easily anticipated without any detailed analysis ?

3. **[3 pts]** The power spectrum of a wide-sense stationary random process is $P_x(e^{j\omega}) = 8\frac{1+\cos 2\omega}{5+4\cos\omega}$. Admit that this random process is generated by exciting a causal and stable linear shift-invariant filter with white noise having unit variance. Find the transfer function of the filter and write a difference equation implementing it.

4. **[1 pt]** Comment the following sentence: «The auto-correlation function of an Auto Regressive process of order *p*, AR(*p*), is an infinite-length sequence».

END (of part 1)