L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2022-2023, week 5
PL problems

Problems related to "Peer-to-peer learning/assessment" (P2P L/A)

NOTE: this week (Oct 10), Student "E3" of each group should explain problem P2P Problem 1, and Student "E4" of each group should explain P2P Problem 2. Detailed information on the P2P procedure is available on the dynamic e-mail sent on Sept 22, 2022.

P2P Problem 1

A measure of the center of gravity of the sequence $x[n]$ may be estimated using

$$C_G = \frac{\sum_{n=-\infty}^{+\infty} n\,x[n]}{\sum_{n=-\infty}^{+\infty} x[n]} . \qquad (1)$$

Using the properties of the Fourier Transform, show that this center of gravity measure may also be estimated in the frequency domain using

$$C_G = \frac{j\left.\frac{dX(e^{j\omega})}{d\omega}\right|_{\omega=0}}{X(e^{j\omega})|_{\omega=0}}$$

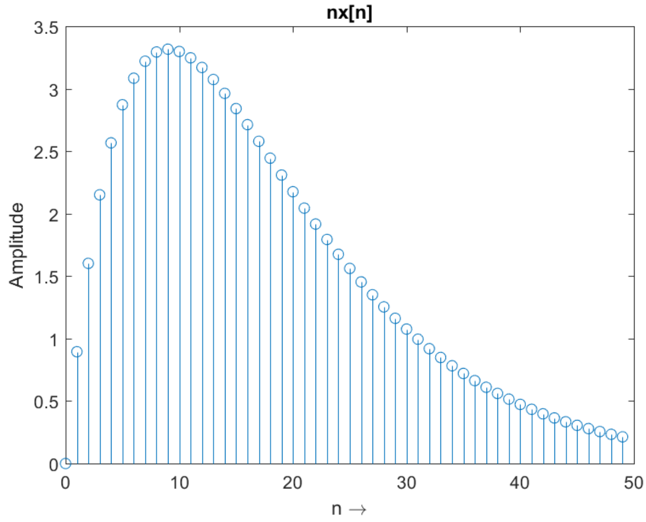
Using this result and considering that $x[n] = a^n u[n]$, show that $C_G = \frac{a}{1-a}$.

P2P assessment: **3pt /5** if demonstration is complete and without errors

Using this compact equation, find C_G in case $= 0.895$. We take this as our reference result.

Now, using Equation (1), estimate C_G in Matlab/Octave when the number of samples is 50, or 100. Are the numerical results consistent with our reference result ?

Just as a reference, a plot of $nx[n]$ is as follows.

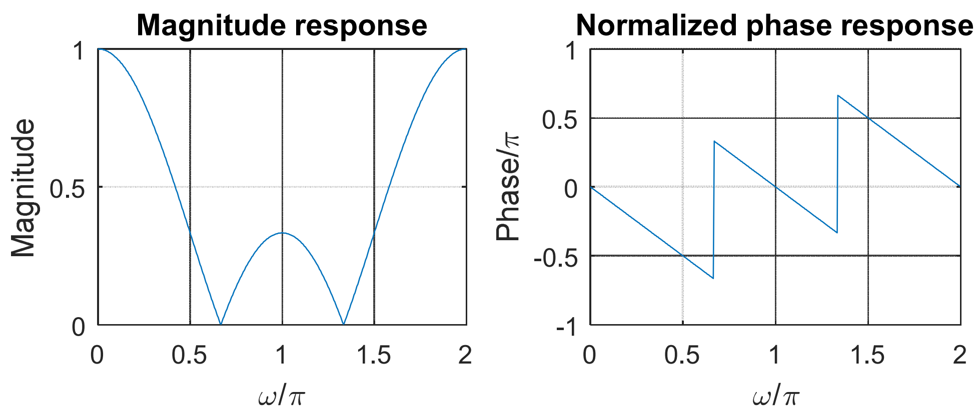


P2P assessment: **2pt /5** if Matlab/Octave code is implemented in such a way as to generate the above figure and if reference $C_G \cong 8.524$ and if the numerical results are shown to be approximately 8.328 and 8.5223.

P2P Problem 2

A discrete-time system consists of a moving average filter of length 3, i.e., its impulse response can be expressed as $h[n] = (u[n] - u[n-3])/3$.

Show in Matlab/Octave that the magnitude and phase representation of the frequency response of this filter, $H(e^{j\omega})$, is as illustrated next.



P2P assessment: **2pt /5** if plots as in the above figure are generated in Matlab/Octave

Show that the compact analytical expressions that reflect those magnitude and phase responses are, respectively:

$$|H(e^{j\omega})| = \frac{1}{3}\frac{\left|\sin\frac{3\omega}{2}\right|}{\left|\sin\frac{\omega}{2}\right|} = \frac{\left|\operatorname{sinc}\frac{3\omega}{2}\right|}{\left|\operatorname{sinc}\frac{\omega}{2}\right|}$$

and

$$\angle H(e^{j\omega}) = -\omega \pm \pi \text{ jumps } \left(\text{when } \frac{\sin\frac{3\omega}{2}}{\sin\frac{\omega}{2}} \text{ switches polarity}\right).$$

P2P assessment: **3pt /5** if demonstration is complete and without errors

Note: in this exercise, we are using the convention $\operatorname{sinc} \alpha = \frac{\sin \alpha}{\alpha}$, in Matlab the convention is $\operatorname{sinc} \alpha = \frac{\sin \pi\alpha}{\pi\alpha}$

Problem extension (not for grading):

If the sampling frequency is 1 kHz, what is the frequency, in Hertz, of a real-valued sinusoid that is exactly blocked by this filter ?

You can demonstrate graphically this using the following Matlab code:

```
FS=1000; TS=1/FS; Freq= ???
t=[0:TS:25E-3];
x=sin(2*pi*Freq*t);
h=ones(1,3)/3;
y=conv(h, x);
figure(1); subplot(2,1,1)
plot(t,x); ylabel('Amplitude'); title('Input')
subplot(2,1,2)
plot(t,y(1:length(x)))
xlabel('Time (s)'); title('Output')
```

Discuss possible transient effects when the signal starts.