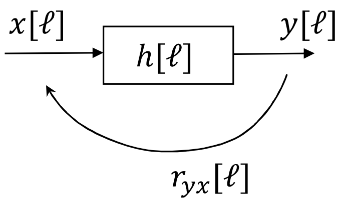
MESTRADO EM ENGENHARIA FÍSICA
EEC4002 - Processamento Estatístico de Sinal

2022-2023 Academic year
Home Assignment (first course module), Due date: November 8, 2022

Exercise 1

Consider a linear and time-invariant (LTI) system whose impulse response is given by $h[\ell]$, and whose input and output sequences are given, respectively, by $x[\ell]$ and $y[\ell]$. As discussed already in a recent lecture, the cross-correlation between output and input is given by $r_{yx}[\ell] = y[\ell] * x^*[-\ell]$, as the following block diagram illustrates.

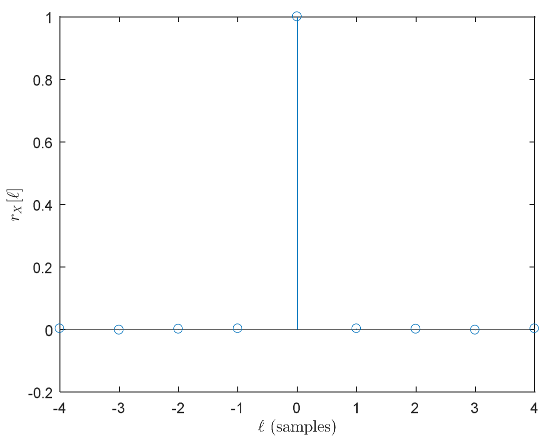


The usual definition of impulse response is the output sequence of a system when it is excited with a special (and simple) deterministic signal: $\delta[\ell]$. In other words, $h[\ell] = y[\ell]|_{x[\ell]=\delta[\ell]}$. In this home assignment, we will estimate the impulse response of the system using as excitation a random sequence whose auto-correlation is the impulse, i.e., $r_x[\ell] = \delta[\ell]$. The random excitation is generated in Matlab as illustrated next.

```
N=1E5;
maxlag=4;

% choose either Gaussian or uniform PDF
x=randn(N,1);
%x=rand(N,1)-0.5;
sigma=sqrt(x.'*x);
x=x/sigma;

% to confirm that rx[ell]=DELTA[ell]
[rx, lag]=xcorr(x, maxlag);
stem(lag, rx)
xlabel('$$\ell$$ (samples)','Interpreter',
'Latex');
ylabel('$$r_{X}[\ell]$$','Interpreter',
'Latex');
```



The impulse response will be estimated using two alternatives: by computing the cross-correlation between output and input, $r_{yx}[\ell] = h[\ell] * r_x[\ell]$, and using the information from the output auto-correlation, $r_y[\ell] = y[\ell] * y^*[-\ell]$. In all cases, we will take as assumption that the discrete-time system is 4$^{\text{th}}$-order, FIR, and minimum-phase. As seen in a recent lecture addressing moving-average (MA) random processes, finding the impulse response in this context, requires that factorization of the power spectrum.

In the Matlab command file `SSP_HA_Oct2022.m`, each group of two Students will find the ground-truth impulse response assigned to each group. This Matlab code also generates the above random excitation all groups should use. In this exercise each group should:

- **a)** obtain $y[\ell]$, $r_{yx}[\ell]$, and obtain a first estimate of $h[\ell]$;
- **b)** obtain $r_y[\ell]$, perform spectral factorization and obtain a second estimate of $h[\ell]$;
- **c)** find the error between the first estimate and the ground-truth, and between the second estimate and the ground-truth; and conclude on which one is more accurate;

d) find if the conclusions to the previous question hold (or not) if, instead of being Gaussian, the PDF of the random excitation is uniform.
NOTE: this PDF change is also included in `SSP_HA_Oct2022.m`

Exercise 2

Determine the power spectrum of a zero-mean WSS process $x[n]$ whose autocorrelation function is:

$$r_x[\ell] = a^{|\ell|} \quad , |a| < 1.$$

Exercise 3

The impulse response of a discrete-time shift-invariant system is $h[n] = a^n u[n]$. If zero-mean white noise with average power σ_x^2 is injected at the input, find the power spectrum of the random signal at the output of the system.

Exercise 4

Find the average power and autocorrelation sequence of the signal at the output of an ideal low-pass filter with cutoff frequency ω_c, when its input is zero-mean white noise with average power σ_x^2.

Exercise 5

A linear shift-invariant filter is used to filter unit variance white noise in order to generate a random process having a power spectrum of the form

$$P_x\left(e^{j\omega}\right) = \frac{5 + 4\cos 2\omega}{10 + 6\cos\omega}.$$

a) Find $P_x(z)$ of the random sequence and factorize it as $P_x(z) = H(z)H(z^{-1})$.

b) Determine the impulse response of the LSI filter.