L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2022-2023, week 11
(not mandatory) P2P exercises

Topic: "Peer-to-peer learning/assessment" exercises addressing sampling the DTFT, and the relationship between the linear convolution and the circular convolution.

Problems related to "Peer-to-peer learning/assessment" (P2P L/A)

NOTE: these area suggested (not mandatory) exercises.

P2P Exercise 1

In this problem, we find the discrete-time Fourier Transform (DTFT) of a finite-length sequence and verify the condition for a uniform sampling of the DTFT (which is the Discrete Fourier transform -DFT) to represent exactly that sequence.

Build a finite-length sequence using the following Matlab code:

```
n=[0:3];
x(1+n)=1+n;
x(7-n)=x(1+n);
figure(1); stem([0:6], x)
axis([-0.5 10 0 5])
xlabel('n (samples)'); ylabel('Amplitude');
```

a) Find the DTFT of the finite-length sequence, i.e. $X(e^{j\omega}) = FT\{x[n]\}$.

Note: the solution should be: $X(e^{j\omega}) = e^{-j3\omega}(6\cos(\omega) + 4\cos(2\omega) + 2\cos(3\omega) + 4)$

b) Show that the analytical solution in **a)** is consistent with the numerical solution in Matlab by executing the following code (which you need to complete):

```
[X W]=freqz(x, 1, 512, 'whole');
figure(2); plot(W/pi, abs(X));
xlabel('\omega/\pi'); ylabel('Magnitude');
pause
omega=W;
myX= PLEASE COMPLETE HERE ;
hold on
plot(W/pi, abs(myX),'m')
hold off
pause
```

c) We now sample $X(e^{j\omega})$ uniformly on the unit circumference using N points, i.e. we take $Y[k] = X(e^{j\omega})\big|_{\omega=k\frac{2\pi}{N},\ k=0,1,\dots,N-1}$, and we synthesize $y[n] = IDFT\{Y[k]\}$, which is an N-periodic sequence. It has been shown in week09 (lecture, see slides or handwritten notes) that, in this case, the relationship between the periodic sequence, $y[n]$, and the finite-length sequence, $x[n]$, is $y[n] = \sum_{\ell=-\infty}^{+\infty} x[n + \ell N]$.

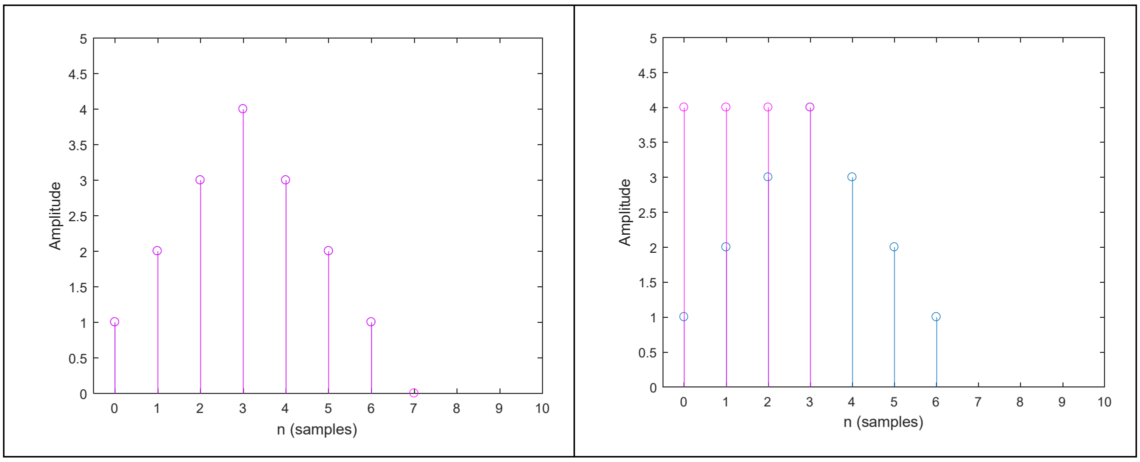
Now, using the following Matlab code (which you need to complete) proceed to sample $X(e^{j\omega})$, as obtained in **a)**, using N points:

```
N=8; k=[0:N-1];
omegak=2*pi*k/N;
Y= PLEASE COMPLETE HERE ;
```

Now, obtain $y[n] = IDFT\{\,Y[k]\}$ for different values of N=8,7,6,5,4. For each one of these four cases, show to your colleagues how is that $y[n]$ compares to $x[n]$ by overlapping the synthesized $y[n]$ on top of $x[n]$ in Figure 1, for example, using the following Matlab code:

```
y=ifft(Y);
figure(1)
hold on
stem(k, real(y), 'm')
hold off
pause
```

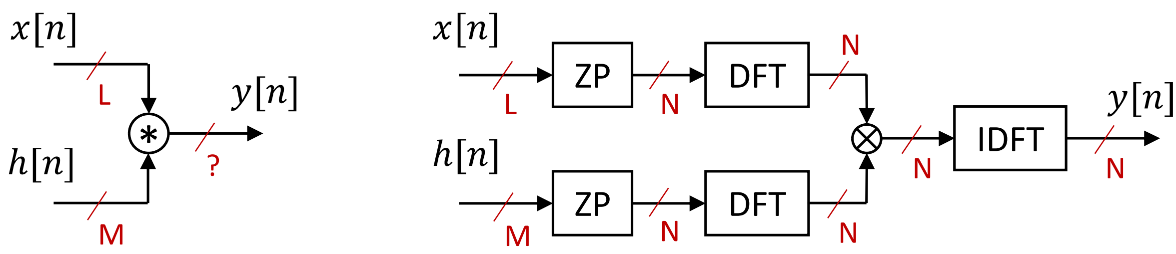
The next figure illustrates two such cases (when N=8, and when N=4). Explain to your colleagues why is that in certain cases $y[n]$ replicates $x[n]$ whereas in other cases that does not happen. Use the relationship $y[n] = \sum_{\ell=-\infty}^{+\infty} x[n+\ell N]$ to explain numerically the different outcomes.



d) By analyzing P2P Exercise 1, and P2P Exercise 2, explain to your colleagues what the relationship is between $e^{-j3\omega}(6\cos(\omega) + 4\cos(2\omega) + 2\cos(3\omega) + 4)$ and $4e^{-j\frac{3\omega}{2}}\cos(\omega)\cos(\omega/2)$, and why. Confirm your answer in Matlab.

P2P Exercise 2

In this problem, we compute the linear convolution between two finite-length sequences (as illustrated next, on the left-hand side), one of them having length L, and the other one having length M, and compute the circular convolution between those sequences by multiplying their DFT of length N (as illustrated next, on the right-hand side). We study the condition to be met for the result of the linear convolution to be exactly the same as the result of the circular convolution.



Create two finite-length sequences, $x[n]$ and $h[n]$, using the following Matlab code:

```
x=[1 1 1 1]; h=x;
ylin=conv(x,h);
figure(1)
stem([0:6], ylin)
axis([-0.5 10 0 5])
xlabel('n (samples)'); ylabel('Amplitude');
pause
```

a) Find the DTFT of $x[n]$, i.e. $X(e^{j\omega}) = FT\{x[n]\}$.

Note: the solution should be:

$$X(e^{j\omega}) = 4e^{-j\frac{3\omega}{2}}\cos(\omega)\cos(\omega/2) = 2e^{-j\frac{3\omega}{2}}(\cos(\omega/2) + \cos(3\omega/2))$$

b) Show that the analytical solution in **a)** is consistent with the numerical solution in Matlab by executing the following code (which you need to complete):

```
figure(2);
[X W]=freqz(x, 1, 512, 'whole');
plot(W/pi, abs(X));
xlabel('\omega/\pi'); ylabel('Magnitude');
pause
omega=W;
myX= PLEASE COMPLETE HERE ;
hold on
plot(W/pi, abs(myX),'m')
hold off
pause
```

c) Now, clarify the meaning of "ZP" in the above block diagram, and compute the result of the circular convolution between $x[n]$ and $h[n]$, using the following Matlab code (which you need to complete):

```
N=8; n=[0:N-1];
% zero padding
x=[x zeros(1, N-length(x))];
h=[h zeros(1, N-length(h))];
X=fft(x); H=fft(h);
Y= PLEASE COMPLETE HERE ;
ycirc=ifft(Y);
figure(1)
```

```
hold on
stem(n, real(ycirc), 'm')
hold off;
```

This Matlab code also compares the result of the circular convolution with the result of the linear convolution by overlapping both graphical representations. Try different values of N=8,7,6,5,4 and, for each one of these four cases, show to your colleagues why the requirement for the result of the circular convolution to equal the result of the linear convolution is met, or not.

d) By analyzing P2P Problem 1, and P2P Problem 2, explain to your colleagues what the relationship is between $e^{-j3\omega}(6\cos(\omega) + 4\cos(2\omega) + 2\cos(3\omega) + 4)$ and $4e^{-j\frac{3\omega}{2}}\cos(\omega)\cos(\omega/2)$. Confirm your answer in Matlab.