
COMPUTER SECURITY

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Cryptography: more advanced topics

Long¹ texts' encipherment: operation modes

Base method

- divide P , plaintext, in parts of equal size (blocks) $P = P_1 P_2 \dots$
 - pieces could be of 1 b, 1 B, 8 B (typical), 16 B (typical)...
- encipher each part separately by one of the methods:
 - stream²
 - each part uses a different key: $C = E_{K_1}(P_1) E_{K_2}(P_2) \dots$
 - or, for simplicity: $C = K_1(P_1) K_2(P_2) \dots$
 - in practice, encipher function usually is (bitwise) plain XOR, \oplus !
 - block
 - each part uses same key: $C = K(P_1) K(P_2) \dots$
 - “mix” of previous
 - same key for each part acts as successive different key

1 See that, in practice, almost any text is "long"! At least regarding symmetric cryptography.

2 PT: contínuo, sequencial

Rationale for "operation modes"

- stream
 - Pro: most secure (even, provable secure with *One-time pad*)
 - Con: very long, one-time usable (random) key
- block
 - Pro: single (random) key
 - Con: same plaintext, same ciphertext
 - if $P_1 = P_2$, then $C_1 = C_2$ [FIG]
- mixed
 - Pro: single (random) key
 - Con: added complexity; possible vulnerability to undetectable modifications of ciphertext!

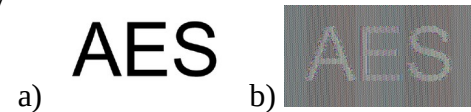


Fig. a) original picture;
b) enciphered with AES 256b,
ECB mode

Some operation modes

Stream method

- Most common: $E = D = \text{plain XOR}, \oplus$
 - e.g. $C_1 = P_1 \oplus K_1$; $P_1 = C_1 \oplus K_1$
- Observation:
 - K_i should be random and not be reused
- Notation:

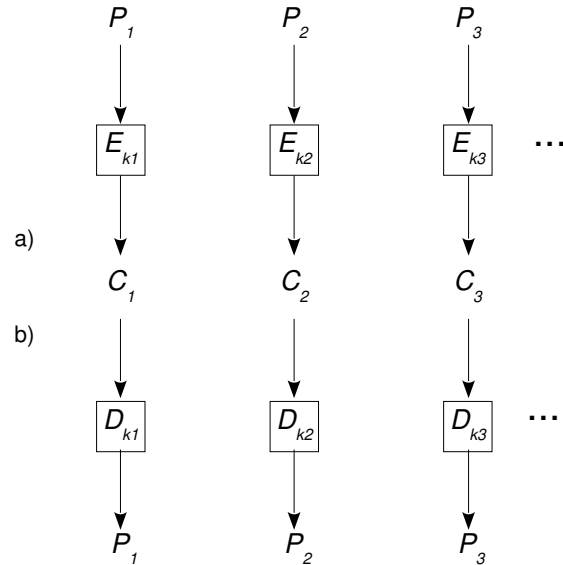
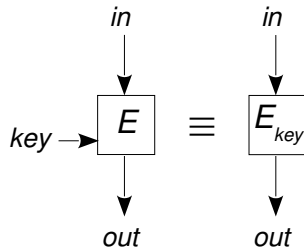


Fig. Use of plain stream method: a) enciphering; b) deciphering.

...“Long” texts' encipherment: operation modes...

Block method

- *ECB, Electronic Code Book*
- Some properties:
 - padding of last block
 - parallelizable en/deciphering
- Formulas:
 - $C_i = E_k(P_i)$, $i > 0$
 - Write the decipherment formula. :-)

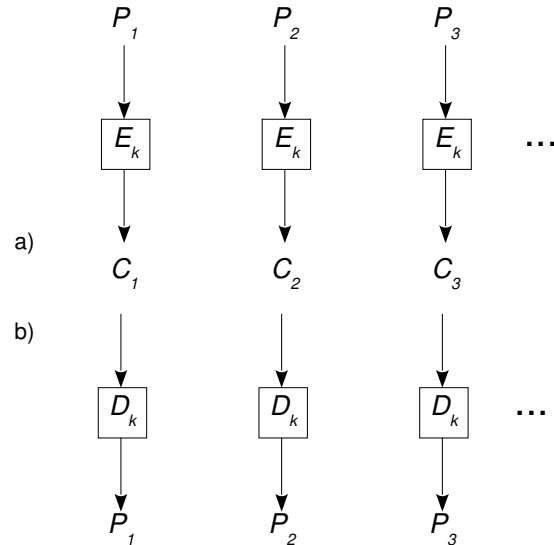


Fig. Use of (plain) block method: a) enciphering; b) deciphering.

...“Long” texts' encipherment: operation modes...

“Mix” method: CTR

- CTR, Counter Mode
- Some properties:
 - IV^1 , initialization vector (random+counter)
 - no padding
 - parallelizable en/deciphering
- Formulas:
 - Write the formulas for the encipherment (C as function of P) and vice versa.

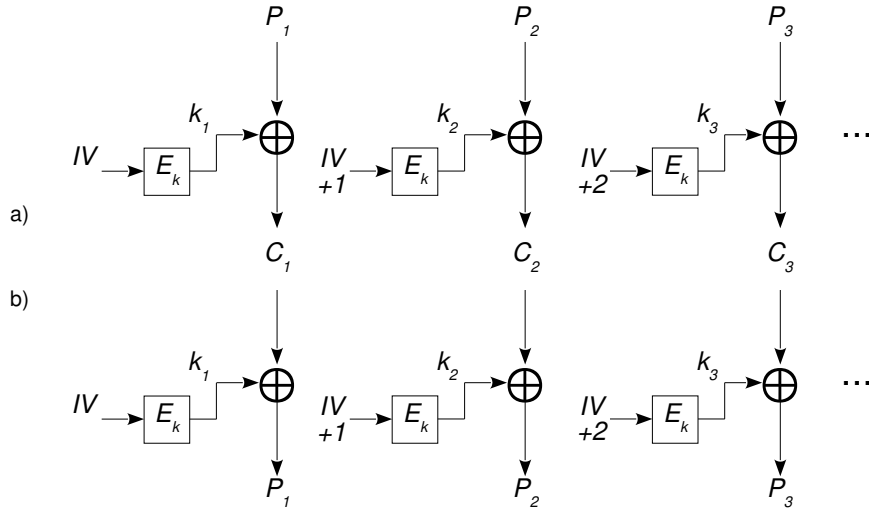


Fig. Use of “mixed” method CTR: a) enciphering; b) deciphering. (Notice the virtual keys k_i .)

1 public value that, as a rule, should be random

...“Long” texts' encipherment: operation modes...

“Mix” method: CFB

- CFB, Cipher Feedback
- Some properties:
 - IV, initialization vector
 - no padding
 - not parallelizable enciphering;
parallelizable deciphering
- Formulas:
 - $C_0 = IV$;
 - $C_i = P_i \oplus E_k(C_{i-1})$, $i > 0$
 - Write the decipherment formula.

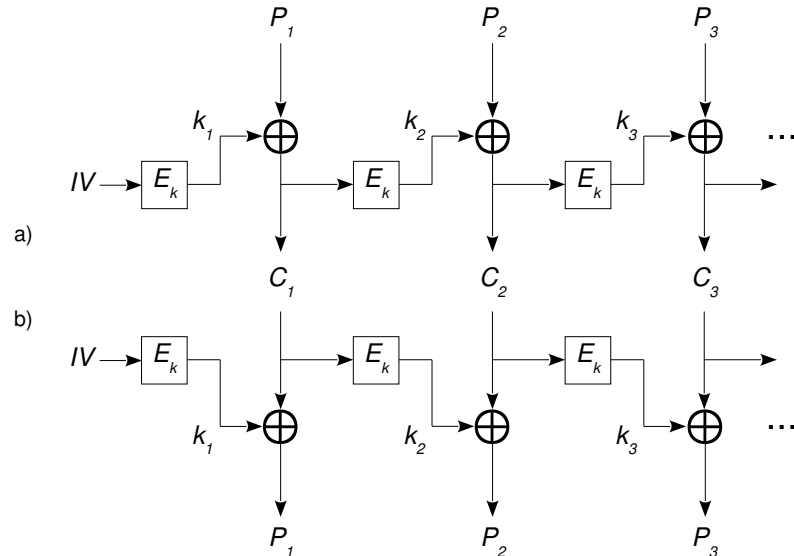


Fig. Use of “mixed” method CFB: a) enciphering; b) deciphering.
(Notice the virtual keys k_i .)

...“Long” texts' encipherment: operation modes...

“Mix” method: OFB

- OFB, Output Feedback
- Some properties:
 - IV, initialization vector
 - no padding
 - not parallelizable en/deciphering, but successive $E_k^i(IV)$ can be done in advance
- Formulas:
 - $C_i = P_i \oplus E_k^i(IV)$, $i \geq 0$
 - Write the decipherment formula.

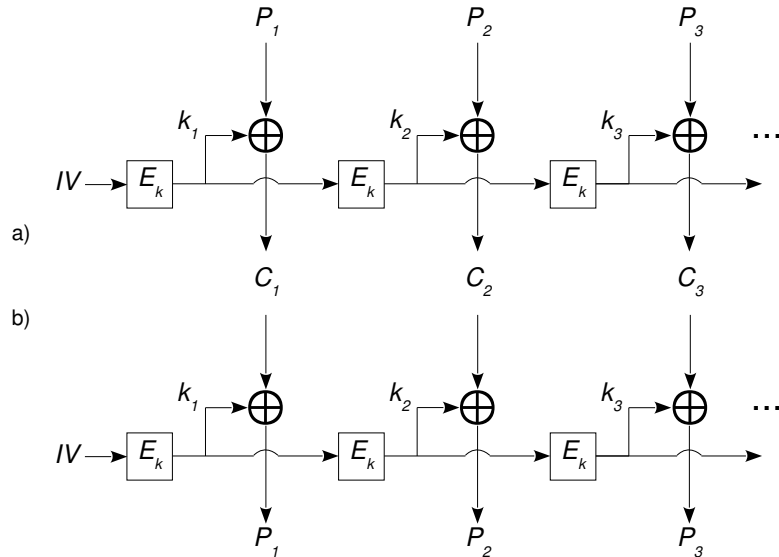


Fig. Use of “mixed” method OFB: a) enciphering; b) deciphering.
(Notice the virtual keys k_i .)

...“Long” texts' encipherment: operation modes...

“Mix” method: CBC

- *CBC, Cipher Block Chaining*
- Some properties:
 - *IV*, initialization vector or explicit initialization by (phony) 1st block!
 - padding
 - not parallelizable enciphering; parallelizable deciphering
- Formulas:
 - $C_0 = IV$; $C_i = E_k(P_i \oplus C_{i-1}) \quad i > 0$
 - Write the decipherment formula.

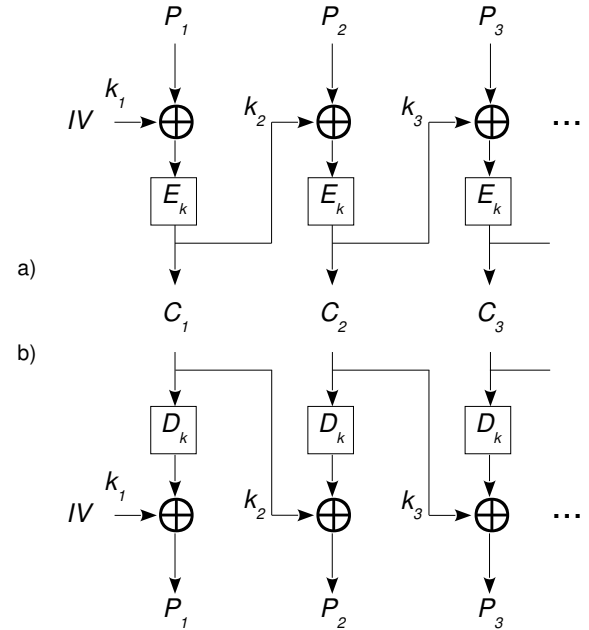


Fig. Use of “mixed” method CBC: a) enciphering; b) deciphering (Notice the virtual keys k_i .)

..."Long" texts' encipherment: operation modes...

Shortcomings of "mixed" block methods:

- some "mixed" methods are vulnerable to modifications of ciphertext
- so, some type of integrity protection must be added to the confidentiality protection: authenticated encipherment modes (*see later*)
 - (external) combination of protective techniques
 - intrinsic combination (authenticated modes)

..."Long" texts' encipherment...

Padding

Need

- size of plaintext varies (just hardly ever is multiple of block size)
 - so, final block might need¹ padding!
 - but, "casual" padding might open an attack path (*see ahead*)!
- harden message deciphering and traffic analysis²
 - by obscuring the size (and content) of ciphertext
 - e.g. avoiding short messages' attack on RSA³
 - e.g. avoiding deterministic ciphering's attack⁴

1 some "modes of operation" do not need padding... why?

2 interception and examination of (ciphered or not) communications to deduce information (e.g. from patterns)

3 asecuritysite.com/encryption/crackrsa2

4 en.wikipedia.org/wiki/Deterministic_encryption

..."Long" texts' encipherment: Padding...

Padding schemes

- several schemes (bit padding or, more usually, byte padding)
 - shared-key cryptography
 - e.g. PKCS¹ #5², #7³ (enciphering) [FIG]
 - one-way cryptography
 - e.g. RFC 6234 (SHA-1, SHA-256) [FIG]
 - e.g. SHA3 (sponge) [FIG]
 - public-key cryptography
 - e.g. PKCS #1 v2 (RFC 8017)
 - RSA's PKCS1-v1_5 [FIG]
 - RSA's OAEP, Optimal Asymmetric Encryption Padding [FIG]
 - Exercise (after analyzing picture): what about deciphering?... does receiver need *seed* and *L*?...

1 Public Key Cryptography Standards, devised and published by RSA Security LLC since the 1990s

2 PKCS #5: Password-Based Cryptography - from a password, get a (symmetric) key for a following symmetric encipherment.

3 #7 padding just extends 8B block #5 padding to 16B (128b) blocks

...”Long” texts' encipherment: **Padding examples (figs)...**

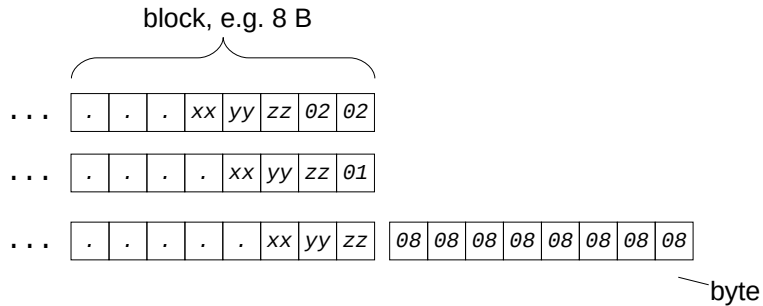


Fig: Shared-key cryptography padding: examples for PKCS #5 (8B blocks); #7 will be similar, but appropriate to 16B blocks.

...”Long” texts' encipherment: Padding examples (figs)...

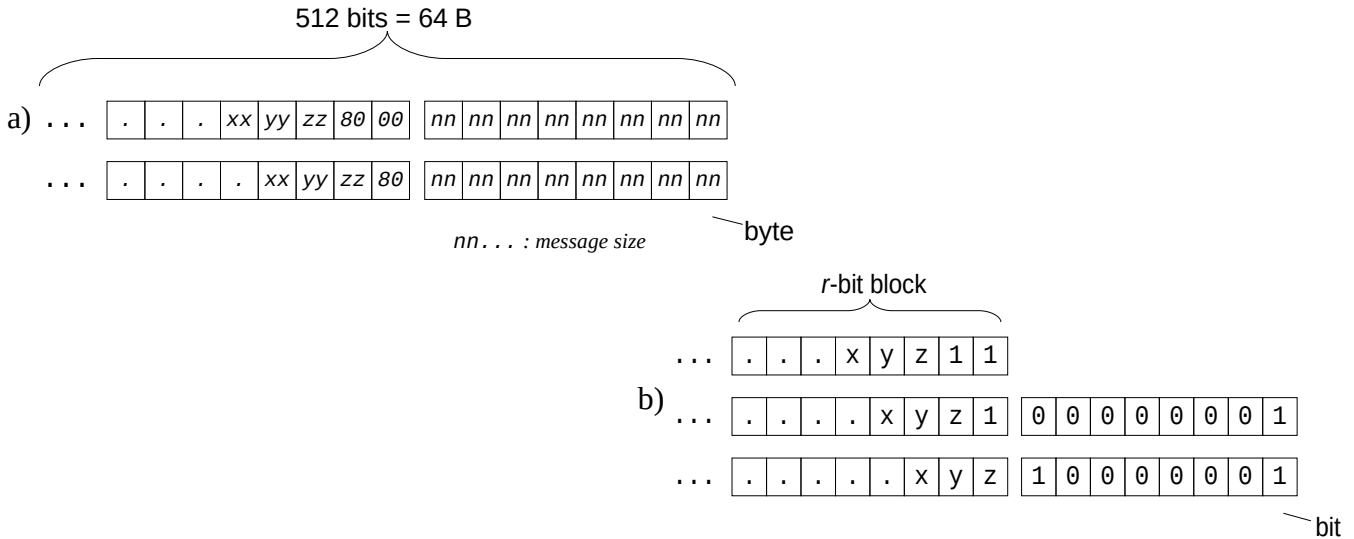


Fig: Instances of one-way cryptography padding:
 a) RFC 6234 padding: (SHA1, SHA256...) - sequence of *nns* is message size;
 b) Sponge multirate padding: 10^*1 (*r* is the number of bits of input block).

...”Long” texts' encipherment: Padding examples (figs)...

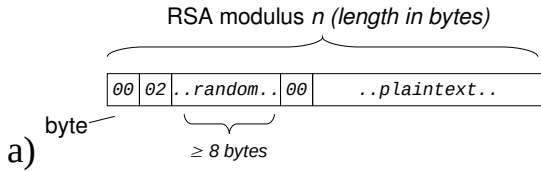


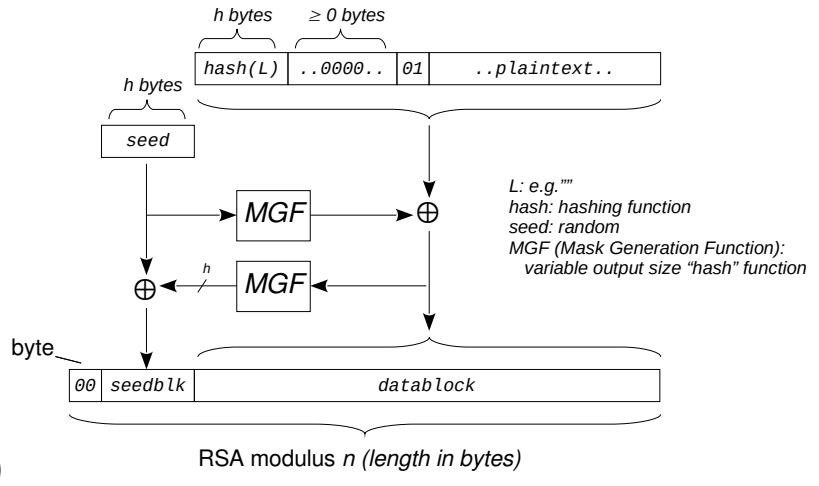
Fig: RSA padding:

a) PKCS1-v1_5 ;

b) OAEP, Optimal Asymmetric Encryption Padding

(*L*, Label, can be empty string; *hash*: hashing function; *seed* must be random; *MGF*, Mask Generation Function, produces pseudorandom variable size strings).

After padding, RSA enciphering proceeds with final data being treated as of *n*-byte hex number.



..."Long" texts' encipherment: Padding...

Attack examples

- length extension: one-way cryptography, MAC (if = $h(K||P)$)
 - if $hash(P1) = hash(IV, P1) = hash(hash(IV), P1)$
 $hash(P1||P2) = hash(P1, P2) = hash(hash(P1), P2)$
 - SEED Lab!
- padding oracle: two-way cryptography, CBC mode
 - if attacker can keep testing decipherment with crafted ciphertext
 - if deciphering error code says explicitly "*invalid padding*" instead of a general "*decryption failed*"
 - CBC: $P_i = D_k(C_i) \oplus C_{i-1}$ $i > 0$
 - a byte/bit change in C_{i-1} affects corresponding byte/bit in P_i
 - starting from last C_i block (where padding is), keep changing last byte until padding is valid; then repeat for previous bytes
 - see [FIG] (PKCS #5, #7 padding)

...”Long” texts’ encipherment: Padding...

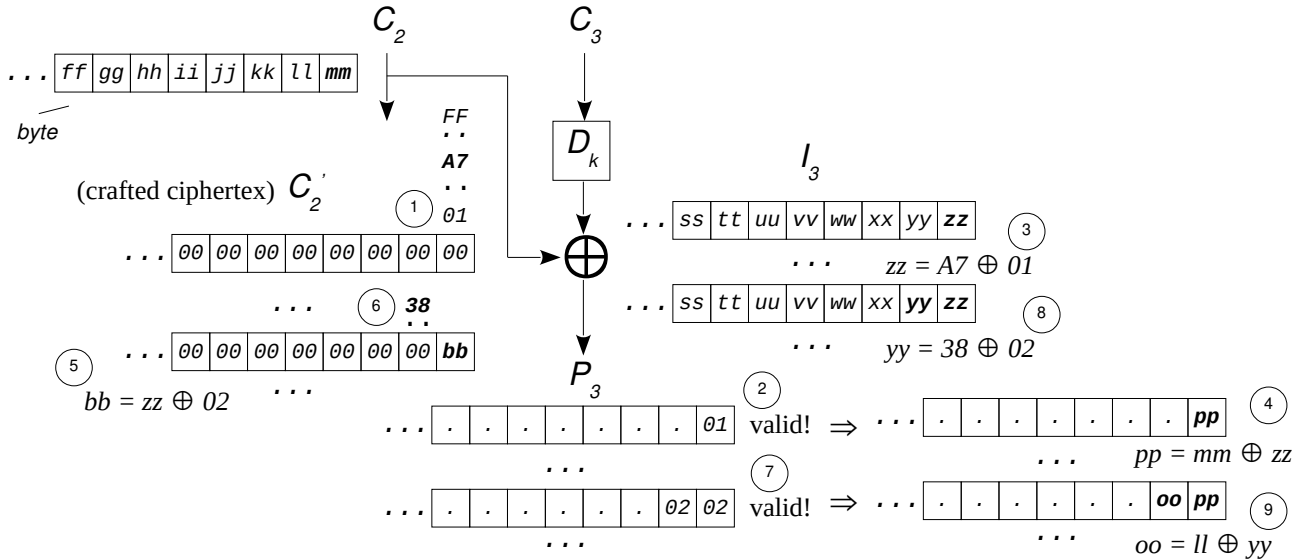
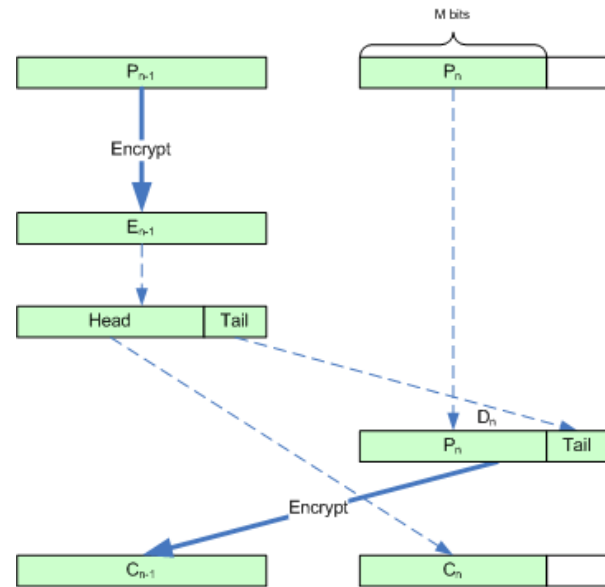
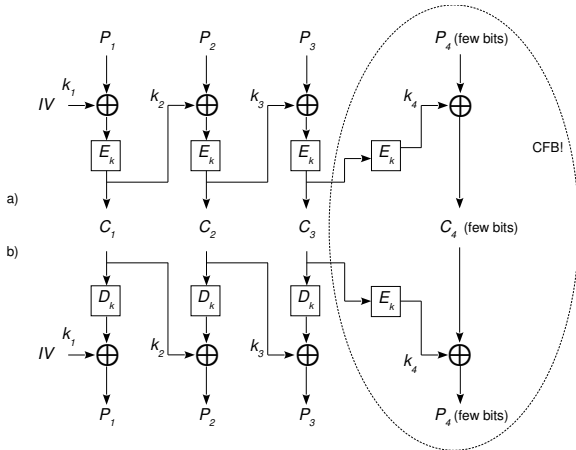


Fig. Padding oracle attack procedure for PKCS #5, #7 padding. C_3 is last cipher block.

...”Long” texts' encipherment: Padding...

Real need for padding?

- avoidance:
 - ciphertext stealing [FIG in Wikipedia]
 - residual block termination [FIG]
- will it be worth the trouble?...



One-way cryptography

Motivation

- «Hash functions are everywhere in cryptography — everywhere!»¹

Applications of one-way functions

- data integrity protection²
 - P public: $F = h(P)$ is characteristic of P
- confirmation of knowledge
 - P secret: presenting public and preset $F = h(P)$ later proves knowledge of P
- key derivation
 - known $k1$, $k2 = h(k1)$ is new key that does not compromise $k1$!
- pseudo-random number generation
 - $seed$ secret: $h^n(seed)$ is apparently random for any successive n
- ...

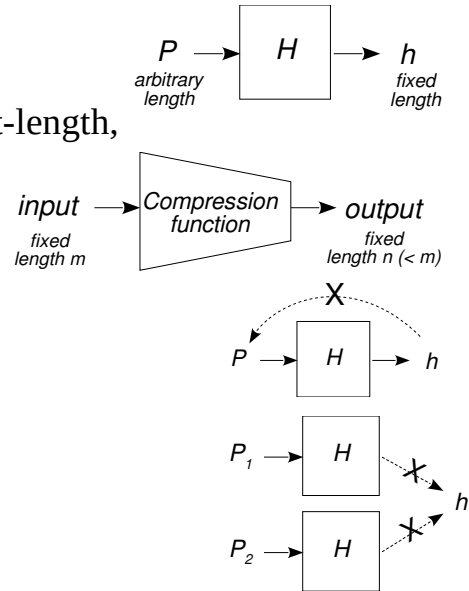
¹ *Real-World Cryptography*, D. Wong, Manning, 2021

² hash functions (unkeyed!) are also called MDC (Modification Detection Code) functions

...One-way cryptography...

Definitions¹

- (minimum) **hash** function H ²
 - compression: maps input P of arbitrary finite bit-length, to output h of fixed bit-length
 - ease of computation: for any P
- **compression** function
 - hash function with fixed-size inputs
- **one-way** hash function
 - impractical³ to invert function
- **collision-resistant** hash function
 - impractical to find two inputs with same output



1 *Handbook of Applied Cryptography*, A.J. Menezes et. al., 5th Printing, CRC Press, 2001

2 can use (secret) keys or not...

3 impractical = currently, computationally infeasible

...One-way cryptography...

Simple examples ($P = P_1 P_2 P_3 \dots = P_1 \parallel P_2 \parallel P_3 \dots$)

- (minimum) **hash** function (*in*, $\text{len}(P)$; *out*, $\text{len}(h)$)
 - $h = P_1 \oplus P_2 \oplus P_3 \oplus \dots$, $\text{length}(P_i) = \text{length}(h)$
- **compression** function (*in*: m bits ; *out*: n bits)
 - $\text{out} = (\text{in's first } n \text{ bits}) \oplus (\text{in's last } (m-n) \text{ bits} \parallel (2n-m) \text{ 0 bits})$
- **one-way** hash function (*in*: m bits ; *out*: n bits)
 - $h = P \bmod h$
- **collision-resistant** hash function
 - ?...

Possible constructions of hash functions

- iterated hash functions (e.g. Merkle–Damgård construction)
 - block cipher based hash functions (e.g. Davies-Meyer construction)
 - using existing secure cipher functions
 - customized (e.g. SHA-1)
 - specifically designed “from scratch” for optimized performance
 - modular arithmetic based¹ (e.g. MASH-1)
 - quite few implementations as research interest is low:
 - sluggish relative to customized hash functions, «*embarrassing history of insecure proposals*» (Menezes et al.)
 - sponge constructions (e.g. SHA-3)
 - new paradigm, allowing easy adjustment of output length

¹ ISO/IEC 10118-4:1998, Hash-functions using modular arithmetic

...One-way cryptography (cont.): Iterated hash functions - Merkle–Damgård construction

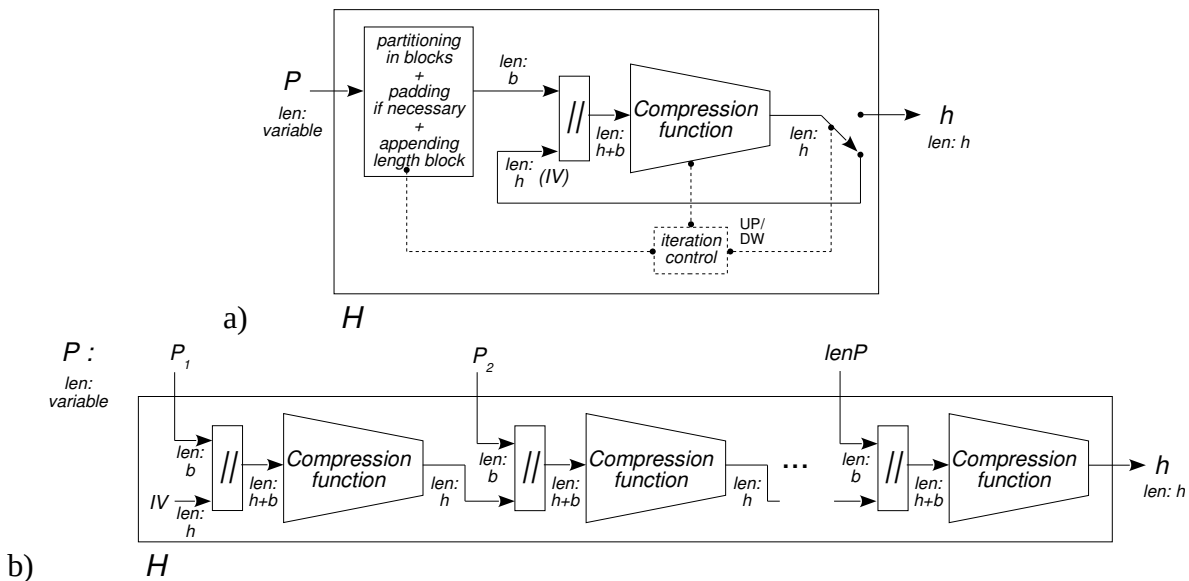


Fig. Two views of the Merkle–Damgård construction: a) software-view ; b) time-view.

...One-way cryptography (cont.): Block cipher based - Davies-Meyer construction

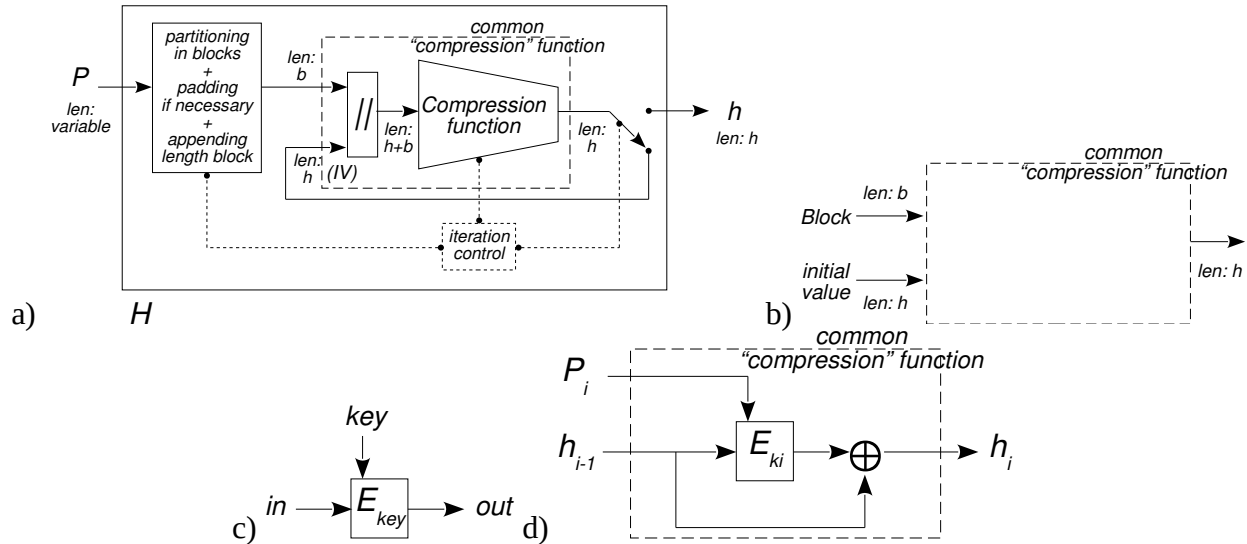
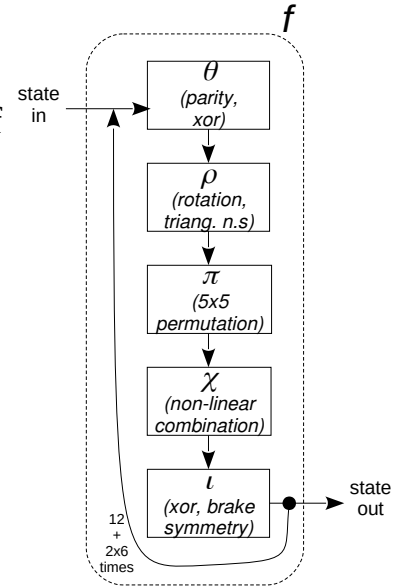
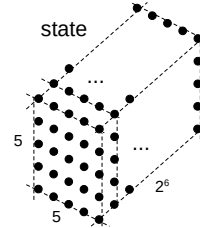


Fig. Davies-Meyer construction for commonly seen "compression functions": a) relation of "common compression" to here presented compression function; b) "compressed" function as block; c) general enciphering snippet - Note that if in is fixed, E_{key} is one-way for mapping $key \rightarrow out!$; d) Davies-Meyer construction.

...One-way cryptography (cont.)

Case study (simplified): SHA-3 (sponge construction)

- besides normal input M , another input parameter can specify length l of output Z [next page FIG]
- padding rule + function Keccak- $f[1600]$ ¹, permutation of $b = r+c$ bits
 - state: $b = 5 \times 5 \times 2^6$ bits = 1600 bits
 - r : bits affected by input ; c : always internal bits
 - permutation: 12 + 2×6 rounds of five steps: θ ρ π χ ι



1 Keccak is pronounced as “ketchak” (keccak.team/keccak_specs_summary.html).

...One-way cryptography (cont.): SHA-3 (sponge construction)

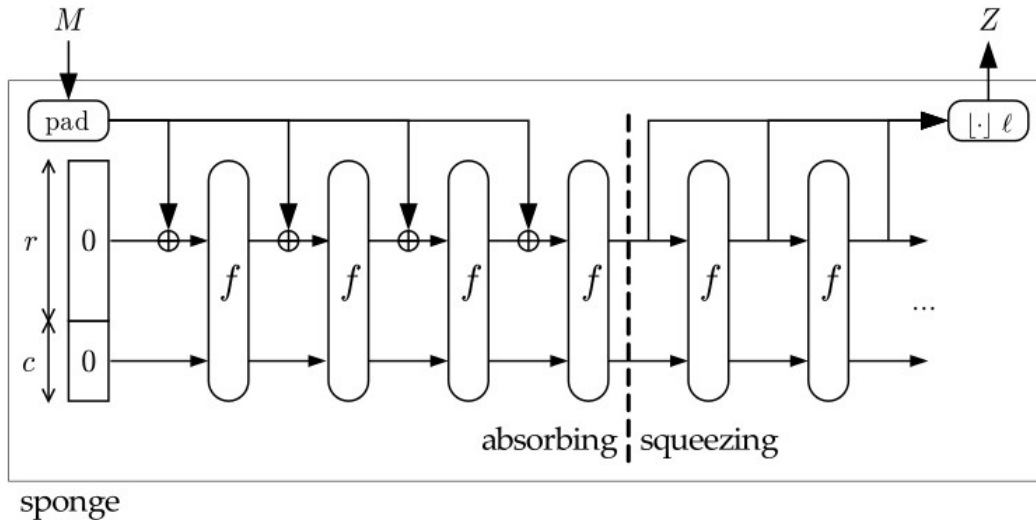


Fig. Sponge construct (time-view): M is input that, after padding, is divided in blocks of r (rate) bits; Z is output of l bits of length (specified by input parameter), concatenation of r bits' blocks; c is capacity, inner state bits, never output. (in keccak.team/sponge_duplex.html)

Overall weaknesses of irreversible systems

Problem:

- The number produced by the hashing operation is usually fixed (finite)
 - So, there **have to be** collisions, in an infinite universe of inputs!
 - Will they be likely or easy to cause?

Answer:

- that depends
 - on the randomness of the values resulting from the operation
 - on the size of those values (number of bits)
 - on the intended application

...One-way cryptography: *Irreversible (cont.)*

Attacks?

- certain: only brute force! (if one can live for enough time...)
 - the intention is to find an entry with a specific result?
 - try 2^n inputs (n , number of bits of *hash*)
- likely: perhaps by using certain curious techniques...
 - the intention is to find two entries with the same result?
 - **birthday attack**: try $\sqrt{2^n} = 2^{n/2}$ inputs for 50% chance of success
 - 2 sets of documents with the same *hash*: one “good” set, one “evil”!¹
- possible: scientifically search for construction weaknesses
 - research, research, research
 - MD5: [MD5 considered harmful today](#)
 - SHA-1: [We have broken SHA-1 in practice](#)
 - ...

¹ Diversity of possibilities for trying different documents are as simple as varying the number of spaces between words...

...One-way cryptography: Irreversible (cont.)

Ideal strength of hash function of n -bit output:

- security is as good as a random oracle with output truncated to n bits
- implies resistance of size:
 - $2^{n/2}$ for strong collision attacks
 - 2^n for weak collision attacks

Example: sponge construction (SHA-3) strength

- with random permutation: as strong as a random oracle
- capacity c determines resistance size:
 - 2^c for both strong and weak collision attacks
 - unfortunately, security is traded for speed, for constant $b (= r+c)$ size
 - higher security (c), lower speed (more r -bit input blocks to process)

Pointers...

- **“Block cipher mode of operation”**, Wikipedia
 - en.wikipedia.org/wiki/Block_cipher_mode_of_operation
- **“The sponge and duplex constructions”**, G. Bertoni, J. Daemen, S. Hoffert, M. Peeters, G. Van Assche, R. Van Keer
 - keccak.team/sponge_duplex.html