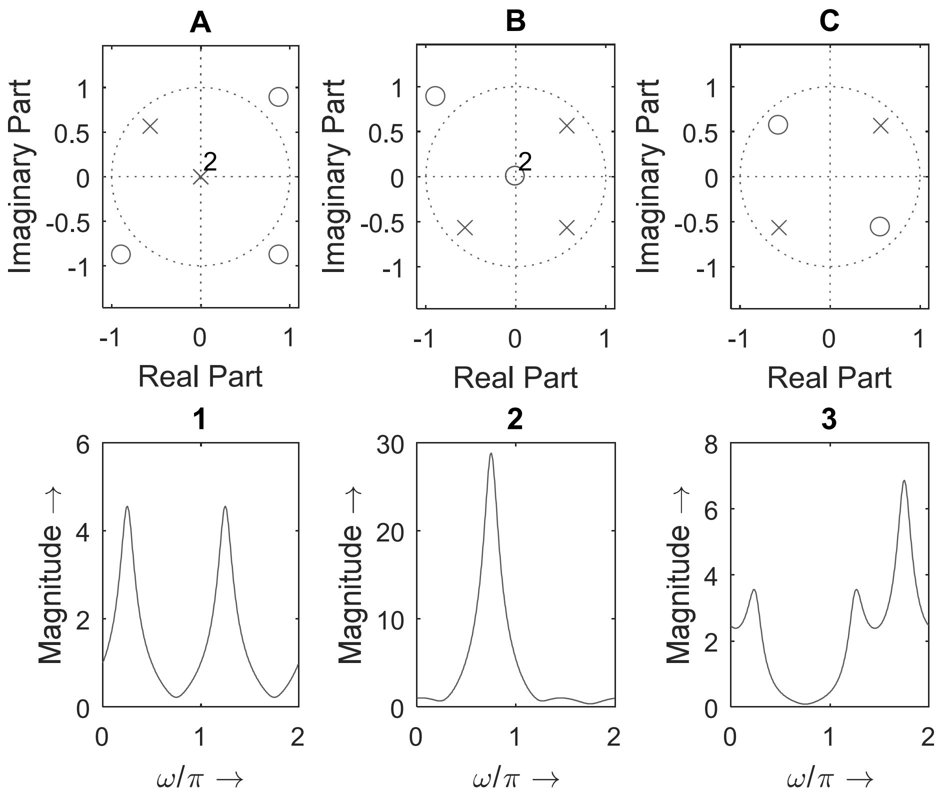
L.EEC/M.EEC
L.EEC025 - Fundamentals of Signal Processing

SECOND EXAM, JANUARY 31, 2023
Duration: 120 minutes, closed book

NOTE: each question (1, 2, 3, 4, 5) ***must*** be answered in a separate sheet; please write your name and Student order number on all sheets, please provide complete answers while trying to minimize paper usage.

1. Three causal discrete-time systems have the illustrated zero-pole diagrams A, B, and C, and the illustrated frequency response magnitudes 1, 2, and 3. Admit that the radius of all poles and zeros is either 0.8 or 1/0.8, and that the angles of all poles and zeros are multiples of π/4.



a) **[1,5 *pts*]** Match each zero-pole diagram (A, B, C) to the corresponding frequency response magnitude (1, 2, 3), and indicate the main supporting arguments.

b) **[1 *pt*]** Explain if a cascade of two of the above systems (AB, AC, or BC) leads to a non-trivial all-pass system. If yes, what is the order of that all-pass system ?

Note: a trivial all-pass system has a transfer function of the form $H(z) = Gz^{-d}$, where G is a constant gain and d is a constant delay.

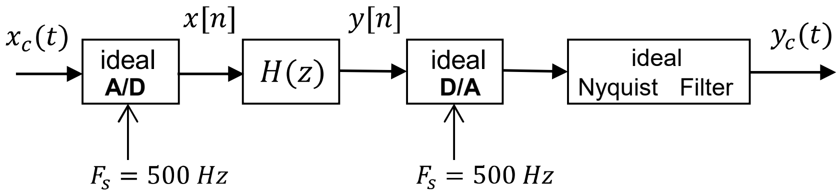
c) **[1 *pt*]** Indicate if it is valid for one of the above systems that $H(z) = \frac{1}{H(jz)}$. Justify.

2. Consider that system C whose zero-pole diagram is represented in Prob. **1** is causal. The radius of all poles and zeros is $r = 0.8$, and their angles are multiples of π/4.

a) **[1,5 *pts*]** Show that except for a constant gain, the transfer function of the system is given by $H(z) = \frac{1+j(rz^{-1})^2}{1-j(rz^{-1})^2}$.

b) **[1 *pt*]** Write a difference equation implementing the system and sketch a corresponding canonic realization structure.

c) **[1 *pt*]** Consider the illustrated analog and causal discrete-time system whose transfer function is as in **b)**. The sampling frequency is 500 Hz. The analog input signal is $x_c(t) = 1 + \cos(875\pi t)$. Notice that an *anti-aliasing* filter does not exist.



Find the sinusoidal frequencies of the discrete-time signal $x[n]$ in the Nyquist range, i.e. in the range $-\pi \le \omega < \pi$. Obtain a compact expression for $x[n]$.

d) **[2 *pts*]** Obtain $y[n]$ and, presuming ideal reconstruction conditions, obtain $y_c(t)$.

Important note: Notice that the impulse response of the discrete-time system is not real-valued and, thus, the discrete-time system output must be separately evaluated for each individual complex exponential at its input.

3. In one of the FPS Labs, the following code was used to service an interrupt-based routine whose relevant C code is as follows (assume that `h[]` represents a vector of N coefficients of the impulse response of a filter, where N is an odd integer, and `x[]` represents a vector of `Nm1=2N-1` input samples):

```
...
x[0]  = (float32_t)(rx_sample_L);
...
switch (myfilter)
{
     case Plain:
        for (i=0 ; i<N ; i++) yn += h[i] * x[i];
        break;
     case Shifted:
        for (i=0 ; i<(N-1) ; )
        {
           yn += h[i] * x[i]; i++;
           yn -= h[i] * x[i]; i++;
        }
        yn += h[i] * x[i];
        break;
     case Upsampled:
        for (i=0, j=0 ; i<N ; i++)
        {
           yn += h[i] * x[j];
           j += 2;
        }
        break;
     default:
        for (i=0 ; i<N ; i++) yn += h[i] * x[i];
}
for (j=(Nm1-1) ; j>0 ; j--) x[j] = x[j-1];

tx_sample_L = (int16_t)(yn);
...
```

a) **[1,5 *pts*]** Explain with words and analytically (i.e. using equations) the operation of this code according to the type of the filter (`Plain`, `Shifted`, `Upsampled`).

b) **[1 *pt*]** Let us admit that $H(z)$ represents the transfer function of the filter whose N impulse response coefficients are stored in vector `h[]`. Explain how would you modify the code such as to implement a filter having $H(z^3)$ as transfer function.

4. Consider that $x[n]$ and $y[n]$ are both *N*-periodic signals and that their DFT are $X[k]$ and $Y[k]$, respectively.

a) **[2 *pts*]** Considering that the $\circledast$ operator denotes circular convolution, demonstrate the property $x[n] \cdot y[n] \overset{DFT}{\longleftrightarrow} \frac{1}{N} X[k] \circledast Y[k] = \frac{1}{N} \sum_{\ell=0}^{N-1} X[\ell] Y[(k-\ell)_N]$, where $n, k = 0,1, \dots, N-1$.

Hint: find the DFT of $x[n] \cdot y[n]$, where $x[n] = \frac{1}{N} \sum_{\ell=0}^{N-1} X[\ell] W_N^{-n\ell}$, and $y[n] = \frac{1}{N} \sum_{m=0}^{N-1} Y[m] W_N^{-nm}$, with $W_\beta^\alpha = e^{-j\frac{2\pi}{\beta}\alpha}$.

Now, consider the following Matlab code which takes advantage of the above DFT property.

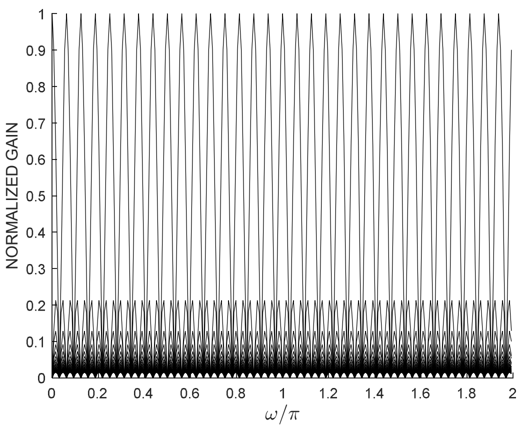
```
x=[j 2 3j 4]; N=length(x);
X=fft(x); Y=X; Y(2:N)=X(N:-1:2);
y=ifft(conj(Y));
Z=cconv(X, conj(Y), N)/N; % cconv() computes the circular convolution
ifft(Z)
```

b) **[1 *pt*]** Without executing the code, find and explain the result of `ifft(Y)`.

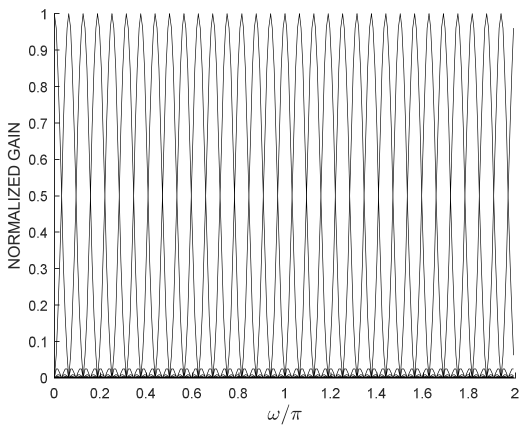
c) **[1 *pt*]** Without executing the code, find and explain the result of `ifft(Z)`.
Note: the Matlab command `cconv(a,b,N)` computes the circular convolution of length N between vectors `a` and `b`

5. The sampling frequency of a real-valued audio signal is 16000 Hz and its spectral contents was analyzed using a sliding FFT with 50% overlap between adjacent FFTs. Two alternative windows were used: Rectangular and Hanning. Just to facilitate visualization, diagrams A and B represent the frequency response magnitude of the <u>even-indexed</u> filters of the FFT filter bank (i.e. for k=0,2,4…). Diagrams C and D represent the spectrograms obtained with all FFT sub-bands and using the two alternative windows.

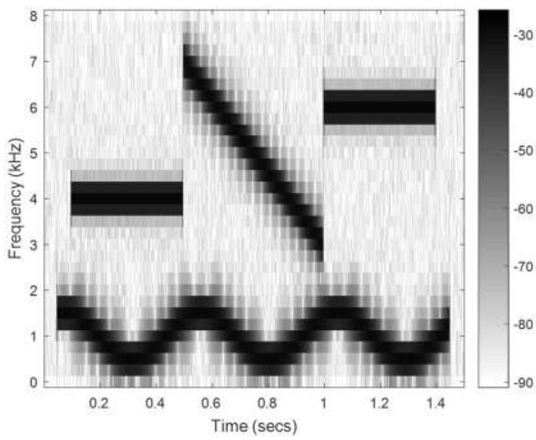
A



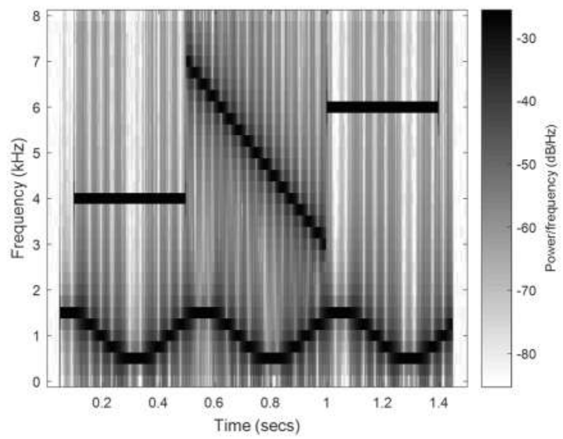
B



C



D



a) **[1,5 *pts*]** The size the FFTs used is a power-of-two number. Using the information in diagrams A and B find what that size is.

b) **[1,5 *pts*]** Explain the impact of a window in a spectrogram representation and match each diagrams A, B to the corresponding spectrogram C, D. Justify.

Note: the blurred effects in the spectrograms reflect the impact of signal processing and not possible printer problems

c) **[1,5 *pts*]** Based on the observation of the diagrams, describe the spectral contents of the signal.

END