FunSP, week01

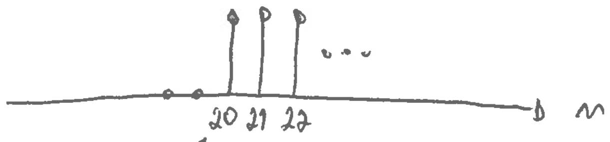
$\boxed{\text{Ex 1}}$ notes:

$$x[n] = u[n-20] - u[n-30]$$

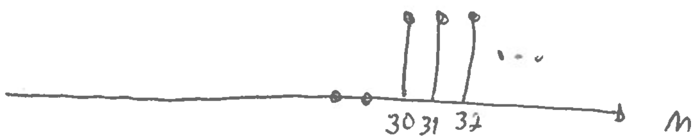
unit step $u[n]$


shifted unit step $u[\underbrace{n-20}_{=0}]$

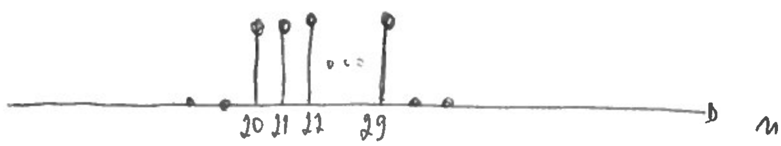
$\therefore\ n = 20$



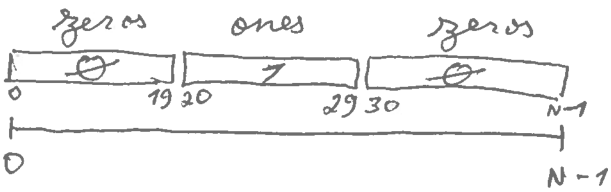

$u[n-30]$




$x[n]$



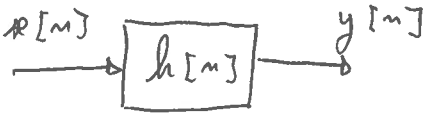

in Matlab:




NOTE: indexing in Matlab starts at 1 (not zero...)

$\boxed{\text{Ex 3}}$ $h[n] = 2^{-n} u[n]$

$x[n] = u[n] - u[n-10]$

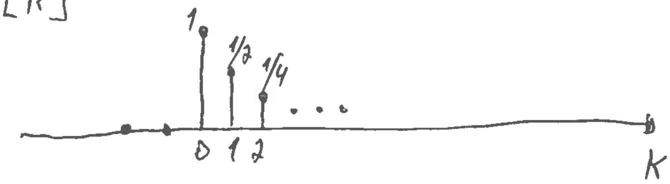



$$y[n] = h[n] * x[n] \triangleq \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]$$ ← we will be using this alternative

$$= x[n] * h[n] \triangleq \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$ ← repeat the exercise at home using this other alternative...

First, we rename the independent variable as $k$ :

$$h[k] = 2^{-k} u[k]$$




Second, we should understand what $x[n-k]$ is considering that $k$ represents independent variable, and $n$ represents a parameter because :

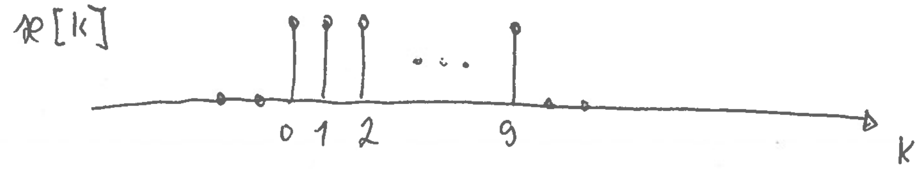
$$y[n] = \sum_{k} \underbrace{h[k]}_{\text{independent variable}}\; x[\underbrace{n}_{\text{parameter}} - \underbrace{k}_{\text{independent variable}}]$$

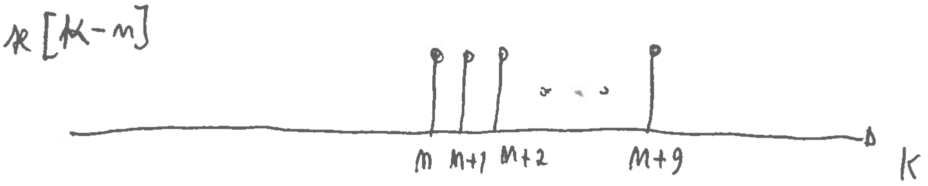
We understand that by noting that :

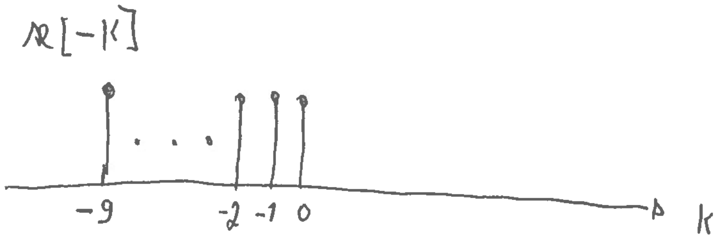
$$x[n-k] = x[-(k-n)]$$

which means that first we have $x[k]$, then $x[k-n]$ and, finally, $x[-(k-n)]$ ; in this process, we also need to understand what $x[-k]$ is with respect to $x[k]$

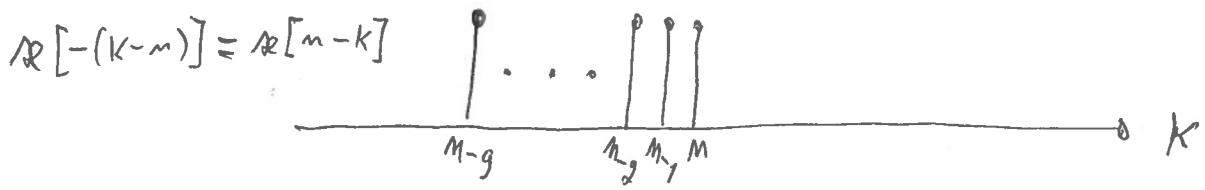
graphically we have:




which is the "mirror" version of $x[k]$ with respect to $k=0$, in the same way, $x[-(k-n)]$ is the "mirror" version of $x[k-n]$ with respect to $k-n=0$, i.e. with respect to $k=n$, thus, we have

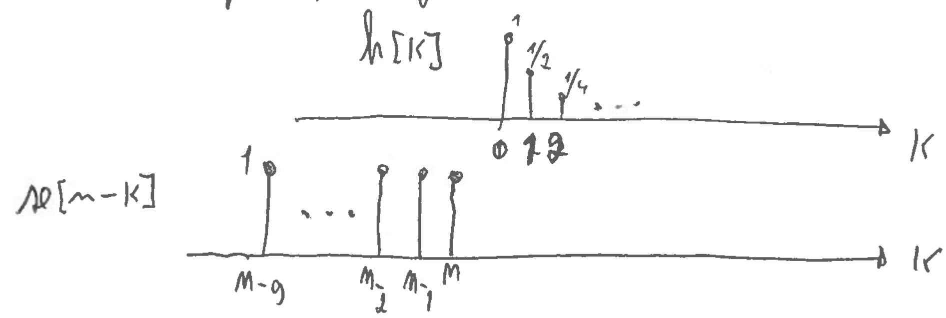



this means that when the $n$ parameter increases, the whole signal moves to the right and, when the $n$ parameter decreases, the whole signal moves to the left

Now, we just need to find, for each value of the $n$ parameter, the result of the element-wise product between the discrete-time sequences $h[k]$ and $x[n-k]$, and add all those products

we should address this by identifying intervals of values of $n$ where the solution is given by the same analytical expression.

For example, if $n<0$ then we have



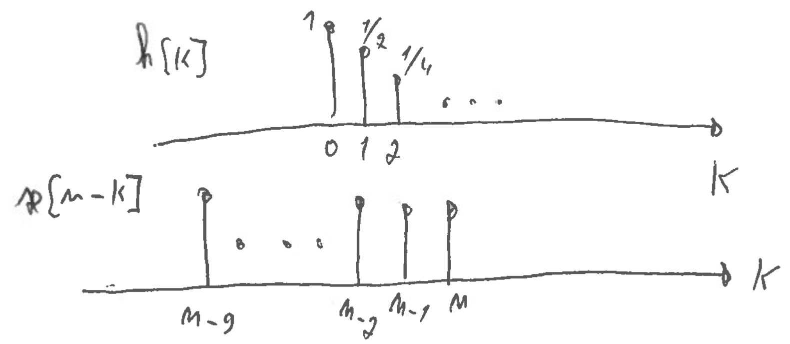

which makes it clear that the element-wise product between the two sequences is always zero, and, thus, the first interval of solutions is:

$\boxed{n<0}$ $\quad y[n]=0$

Next, when $n \geq 0$, then there is an "overlap" between sequences $h[k]$ and $x[n-k]$, however, this overlap is either partial (when $n-9<0$) or total (when $n-9 \geq 0$), which gives rise to two new intervals:

$\boxed{0 \leq n \leq 9}$

could be $<9$




In this case, we have:

$$y[n] = \sum_{k=0}^{n} h[k]\, x[n-k] = \sum_{k=0}^{n} 2^{-k} = \frac{1-2^{-(n+1)}}{1-\frac{1}{2}}$$

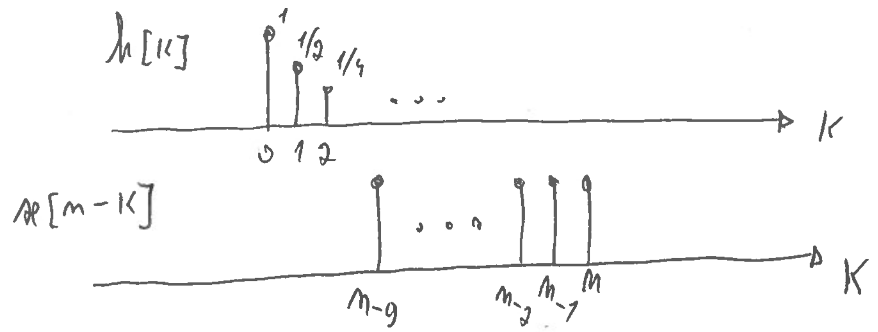
$$= 2\left(1-2^{-n}2^{-1}\right) = 2-2^{-n} = \frac{2^{n+1}-1}{2^{n}}$$

In this process, we should recall (and always keep in mind) the closed-form expression that gives the sum of terms of a geometric series:

$$\boxed{\sum_{k=a}^{b} r^k = \frac{r^a - r^{b+1}}{1-r}}$$

Similarly, when

$\boxed{n \geq 9}$ $\quad y[n] = \sum_{k=n-9}^{n} h[k]\, x[n-k]$ because




and, thus,

$$y[n] = \sum_{k=n-9}^{n} 2^{-k} = \frac{2^{-(n-9)} - 2^{-(n+1)}}{1-\frac{1}{2}} = 2\left(2^{-n}2^{9} - 2^{-n}2^{-1}\right)$$

$$= \frac{2^{10}-1}{2^{-n}}$$

An illustration of this three-interval solution is: