PDSI17FEB2021

$$x[n] = x_e[n] + x_o[n]$$

$$x_e[n] = \frac{x[n] + x^*[-n]}{2} = x_e^*[-n] \xleftrightarrow{\mathcal{F}} \mathcal{Re}\{X(e^{j\omega})$$

$$x_o[n] = \frac{x[n] - x^*[-n]}{2} = -x_o^*[-n] \longleftrightarrow \mathcal{Im}\{X(e^{j\omega}$$

$$x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega})$$

$$\boxed{x^*[n] \longleftrightarrow X^*(e^{-j\omega})}$$

$$x[-n] \longleftrightarrow X(e^{-j\omega})$$

$$x^*[-n] \longleftrightarrow X^*(e^{j\omega})$$

$$\frac{x[n] + x^*[-n]}{2} \longleftrightarrow \frac{X(e^{j\omega}) + X^*(e^{j\omega})}{2}$$

$$= \mathcal{Re}\{X(e^{j\omega})\}$$

$$x[n] * y[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega})\, Y(e^{j\omega})$$

$$x[n] \cdot y[n] \longleftrightarrow \frac{1}{2\pi} X(e^{j\omega}) * Y(e^{j\omega})$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta}) \cdot Y(e^{j(\omega-\theta)})$$

$$x[n]\cdot y^*[n] \longleftrightarrow \boxed{\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\cdot Y^*(e^{j(\theta-\omega)})\, d}$$

$$w[n] = x[n]\cdot y^*[n] \longleftrightarrow W(e^{j\omega}) \triangleq \sum_{n=-\infty}^{+\infty} w[n]e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{+\infty} x[n]\cdot y^*[n]\, e^{-j\omega n}$$

$$W(e^{j0}) = W(1) = \sum_{n=-\infty}^{+\infty} x[n]\, y^*[n]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, Y^*(e^{j\theta})\, d\theta$$

$$y[n] = x[n]$$

$$W(1) = \sum_{n=-\infty}^{+\infty} x[n]\, x^*[n] = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = E$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})\, X^*(e^{j\theta})\, d\theta$$

$$= \frac{1}{2\pi}\int^{\pi} |X(e^{j\theta})|^2\, d\theta = E$$

spectral density of energy