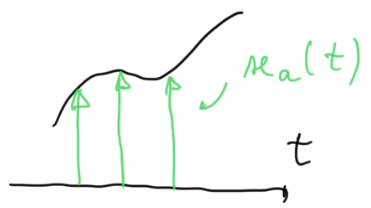
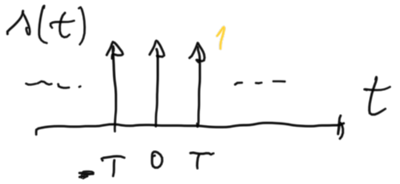
PSFI02NOV2021

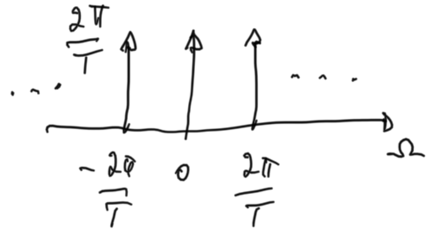
Lecture



$$x_c(t) \xleftrightarrow{\mathcal{F}} X_c(\Omega)$$



$$\xleftrightarrow{\mathcal{F}}$$



$$s(t) = \sum_{m=-\infty}^{+\infty} \delta(t - mT) \longleftrightarrow S(\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\Omega - k\frac{2\pi}{T}\right)$$

$$x_a(t) = x_c(t) \times s(t) \longleftrightarrow X_a(\Omega) = \frac{1}{2\pi} X_c(\Omega) * S(\Omega)$$

$$= x_c(t) \sum_{m=-\infty}^{+\infty} \delta(t - mT)$$

$$= \sum_{m=-\infty}^{+\infty} x_c(mT)\, \delta(t - mT)$$

$$= \frac{1}{\cancel{2\pi}} X_c(\Omega) * \frac{\cancel{2\pi}}{T} \sum_{k} \delta\left(\Omega - k\frac{2\pi}{T}\right)$$

$$X_a(\Omega) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\Omega - k\frac{2\pi}{T}\right)$$

$$X_a(\Omega) = \mathcal{F}\{x_a(t)\}$$

$$= \int_{-\infty}^{+\infty} x_a(t)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x_c(nT)\,\delta(t-nT)\, e^{-j\Omega t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} x_c(nT) \int_{-\infty}^{+\infty} \underbrace{\delta(t-nT)\, e^{-j\Omega t}}_{e^{-j\Omega nT}}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} \underbrace{x_c(nT)}_{= x[n]}\, e^{-j\Omega nT} \qquad \boxed{\underbrace{\Omega}_{\text{rad/s}} T = \underbrace{\omega}_{\text{rad.}}} \qquad \Omega = \frac{\omega}{T}$$

$$= \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\omega n} = X(e^{j\omega}) = X_a(\Omega)\Big|_{\Omega=\frac{\omega}{T}}$$

$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\Omega - k\frac{2\pi}{T}\right)\Bigg|_{\Omega = \frac{\omega}{T}}$$

$$= \boxed{\frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c\left(\frac{\omega - k2\pi}{T}\right)}$$