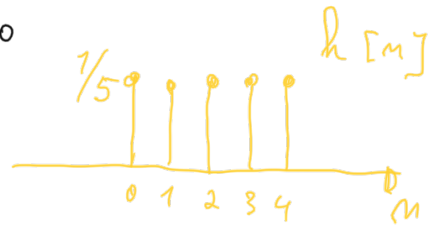


$$y[n] = \frac{1}{5} \sum_{k=0}^4 x[n-k]$$

$$h[n] \triangleq y[n] \Big|_{x[n]=\delta[n]} = \frac{1}{5} \sum_{k=0}^4 \delta[n-k]$$

a)



$$b) H(e^{j\omega}) \triangleq \sum_{n=-\infty}^{+\infty} h[n] e^{-j\omega n} = |H(e^{j\omega})| e^{j\angle H(e^{j\omega})}$$

1. magnitude

2. phase

$$= \underbrace{A(\omega)}_{\text{real-valued}} e^{j \underbrace{B(\omega)}_{\text{real-valued}}}$$

$$H(e^{j\omega}) = \frac{1}{5} \sum_{n=0}^4 e^{-j\omega n} = \frac{1}{5} \frac{1 - e^{-j\omega 5}}{1 - e^{-j\omega}}$$

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$1 - e^{-j5\omega} = e^{j0} - e^{-j5\omega} = e^{-j\frac{5\omega}{2}} \left(e^{j\frac{5\omega}{2}} - e^{-j\frac{5\omega}{2}} \right) = e^{-j\frac{5\omega}{2}} 2j \sin \frac{5\omega}{2}$$

$$1 - e^{-j\omega} = e^{-j\frac{\omega}{2}} \left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} \right) = e^{-j\frac{\omega}{2}} 2j \sin \frac{\omega}{2}$$

$$H(e^{j\omega}) = \frac{1}{5} \frac{e^{-j\frac{5\omega}{2}} 2j \sin \frac{5\omega}{2}}{e^{-j\frac{\omega}{2}} 2j \sin \frac{\omega}{2}} = \frac{e^{-j\frac{5\omega}{2}}}{e^{-j\frac{\omega}{2}}} \frac{\sin \frac{5\omega}{2}}{\sin \frac{\omega}{2}}$$

$$-\frac{5\omega}{2} + \frac{\omega}{2} = -\frac{4\omega}{2} = -2\omega$$

$$H(e^{j\omega}) = \frac{1}{5} e^{-j2\omega} \frac{\sin \frac{5\omega}{2}}{\sin \frac{\omega}{2}} \Rightarrow \left(\frac{1}{5} \frac{\sin \frac{5\omega}{2}}{\sin \frac{\omega}{2}} \right) e^{-j2\omega}$$

$$\text{III } A(\omega) \quad B(\omega) = -2\omega$$

$$= |H(e^{j\omega})| e^{j\angle H(e^{j\omega})}$$

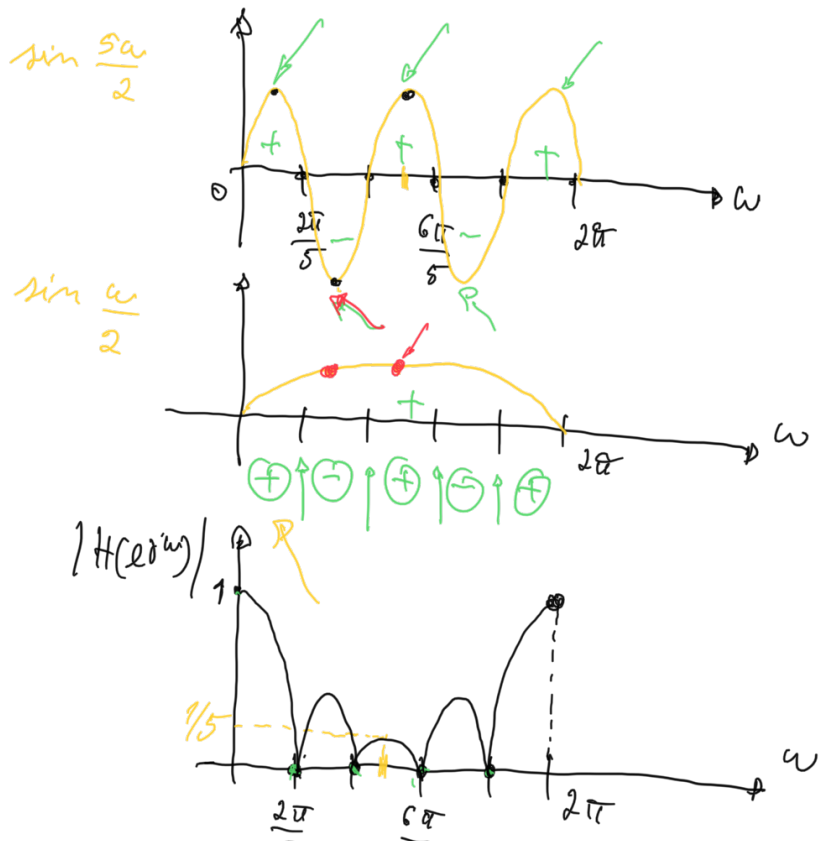
$$|H(e^{j\omega})| = |A(\omega)|$$

$$\angle H(e^{j\omega}) = B(\omega)$$

$$-1 = e^{\pm j\pi}$$

$$\sin \frac{5\omega}{2} = 0 \quad \therefore \frac{5\omega}{2} = k\pi \Leftrightarrow \omega = k \frac{2\pi}{5}$$

$$\sin \frac{\omega}{2} = 0 \quad \therefore \frac{\omega}{2} = l\pi \Leftrightarrow \omega = l 2\pi$$



$$H(e^{j\omega}) = \frac{1}{5} \frac{\sin \frac{5\omega}{2}}{\sin \frac{\omega}{2}} e^{-j2\omega}$$

$$\text{sinc } \alpha = \frac{\sin \alpha}{\alpha}$$

$$\lim_{\alpha \rightarrow 0} \text{sinc } \alpha = 1$$

$$H(e^{j\omega}) = \frac{1}{5} \frac{\sin \frac{5\omega}{2}}{\frac{5\omega}{2}} \cdot \frac{\frac{5\omega}{2}}{\frac{\omega}{2}} \frac{\frac{\omega}{2}}{\sin \frac{\omega}{2}} e^{-j2\omega}$$

$\equiv \text{sinc } \frac{5\omega}{2}$
 $\equiv \frac{1}{\text{sinc } \frac{\omega}{2}}$

$$\text{sinc } \alpha = \frac{\sin \pi \alpha}{\pi \alpha}$$

$$\approx \frac{\text{sinc } \frac{5\omega}{2}}{\text{sinc } \frac{\omega}{2}} e^{-j2\omega}$$

$$H(e^{j\omega}) \Big|_{\omega=0} \approx 1$$

$\pm j\pi$
 $-1 = e$

$$\angle H(e^{j\omega}) \approx -2\omega + \text{--- jumps of } \pi$$

