PDSI23MAR2021

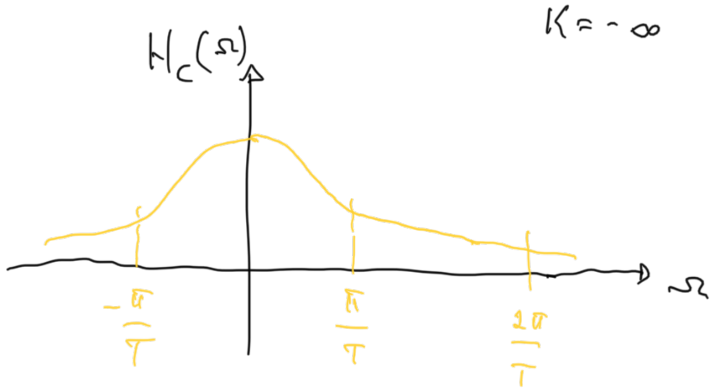
Lecture # 12

I.I.

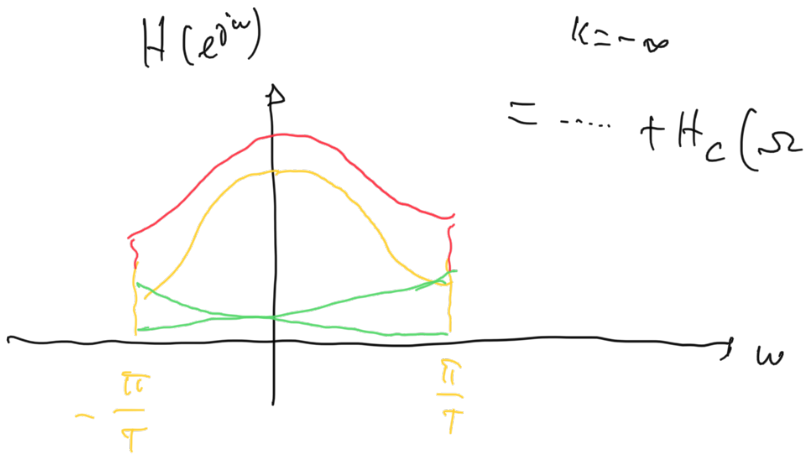
$$h_c(t) \longrightarrow h[n] = T\, h_c(t)\Big|_{t=nT}$$

$$x_c(t) \longrightarrow x[n] = x_c(nT)$$

$$\therefore\; X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X_c\left(\Omega - k\frac{2\pi}{T}\right)\Bigg|_{\Omega=\frac{\omega}{T}}$$



$$\therefore\; H(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} H_c\left(\Omega - k\frac{2\pi}{T}\right)\Bigg|_{\Omega=\frac{\omega}{T}}$$



$$= \ldots + H_c\left(\Omega + \frac{2\pi}{T}\right) + H_c(\Omega) + H_c\left(\Omega - \frac{2\pi}{T}\right)$$

$+ \ldots$

$$H(e^{j\omega}) = H_c(\Omega), \text{ iff}$$

$$|H_c(\Omega)| = 0,$$

$$|\Omega| > \frac{\pi}{T}$$

LAPLACE

$s = \sigma + j\Omega$

$$H_c(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k}$$

$$\therefore$$

$$h_c(t) = \sum_{k=1}^{N} A_k e^{s_k t} u(t)$$

$$h[n] = T h_c(nT) = \sum_{k=1}^{N} T A_k e^{s_k n T} u[n]$$

$$e^{s_k n T} = e^{s_k T n}$$

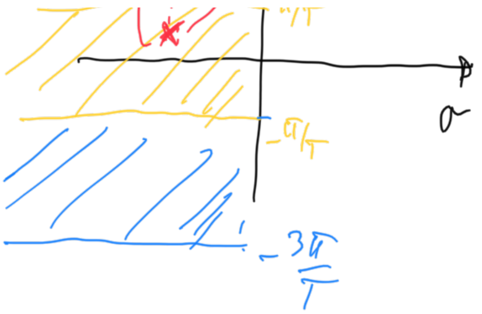
$$H(z) = \sum_{k=1}^{N} \frac{T A_k}{1 - e^{s_k T} z^{-1}}, \quad \text{causal}$$

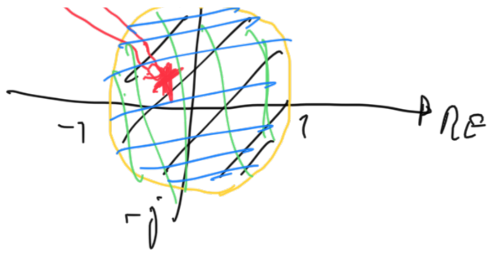
LAPLACE - DOMAIN $\qquad\qquad$ Z - DOMAIN

$$s_k \longrightarrow e^{s_k T} = e^{(\sigma_k + j\Omega_k)T}$$

$$e^{\left(\sigma_k + j\Omega_k + jk\frac{2\pi}{T}\right)T}$$

$$= e^{(\sigma_k + j\Omega_k)T} e^{jk2\pi}$$





B.T.

$$H_c(s) \longrightarrow H(z) = H_c(s)\Big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

B.T.

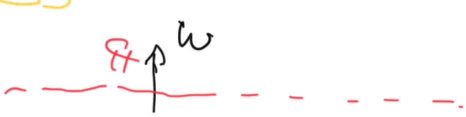
$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$j\Omega = \frac{2}{T}\,\frac{1-e^{-j\omega}}{1+e^{-j\omega}} = \frac{2}{T}\,\frac{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}}-e^{-j\frac{\omega}{2}}\right)}{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}}+e^{-j\frac{\omega}{2}}\right)}$$

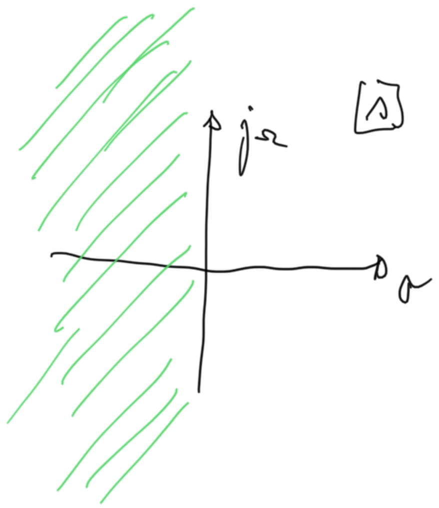
$$= \frac{2}{T}\,\frac{2j\sin\frac{\omega}{2}}{2\cos\frac{\omega}{2}}$$

$$= j\,\frac{2}{T}\,\mathrm{tg}\,\frac{\omega}{2}$$

$$\therefore \quad \boxed{\Omega = \frac{2}{T}\,\mathrm{tg}\,\frac{\omega}{2}} \qquad \therefore \quad \omega = 2\,\mathrm{arctg}\,\frac{\Omega T}{2}$$




Ω
−π


jΩ
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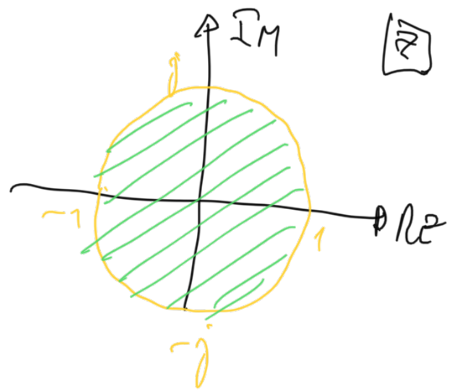
Im
Re
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1
−1