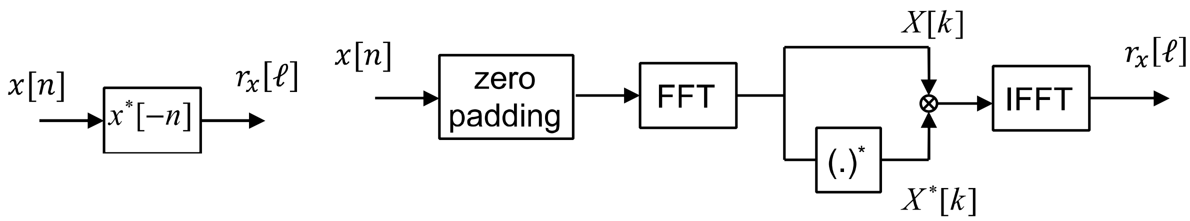
L.EEC025 - FUNDAMENTALS OF SIGNAL PROCESSING

Academic year 2022-2023, weeks 12-13
TP (Recitation) problems

Topics: Computation of the auto-correlation function using the DFT. Time-domain windowing in the frequency-domain. The magnitude spectrum (periodogram) and the spectrogram.

Exercise 1

According to the Wiener-Khintchine theorem, the following two block diagrams represent two possible alternatives to compute the auto-correlation of a signal $x[n]$ that we admit is real-valued and has length L. Consider that both FFT and IFFT have length N, where N is a power of two number.

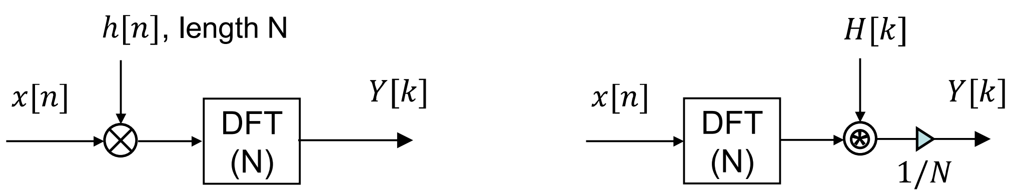


a) Clarify under which condition both alternatives yield the same desired result.

b) Admitting that N=2L, find the lowest value of L above which the second alternative (using FFT) is more advantageous than the first one. Consider just multiplication operations and consider that that the computational cost of a complex multiplication is equivalent to four real multiplications.

Exercise 2

As discussed in recent classes, windowing is an important signal processing step helping to control the leakage phenomenon in DFT-based spectrum analysis. Most often, windowing takes place in the discrete-time domain, as represented on the left-hand side of the following illustration. However, in certain cases, the equivalent operation must be performed in the frequency domain as illustrated on the right-hand side. In this case, it is desired that $H[k]$, the DFT of $h[n]$, has a compact support, i.e. that only a few coefficients are non-zero (why ?).



a) As can be concluded from the developments in the guide concerning this week's Lab, the standard Hamming or Hanning window do not have a compact support. However, the *shifted* Hanning window has a compact support. It is defined as

$$h_{Hann}[n] = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi}{N}\left(n + \frac{1}{2}\right)\right)\right], \quad n = 0,1,\dots,N-1.$$

Find $H[k]$, the DFT of $h_{Hann}[n]$.

b) Using $H[k]$, the signal and N parameter of P2P Problem 1 of week 13, complete the following Matlab code to check the equivalence between windowing in the time domain, and the corresponding operation in the frequency domain.

Hint 1: recall the DFT properties

Hint 2: the circular convolution is implemented in Matlab using `cconv()`

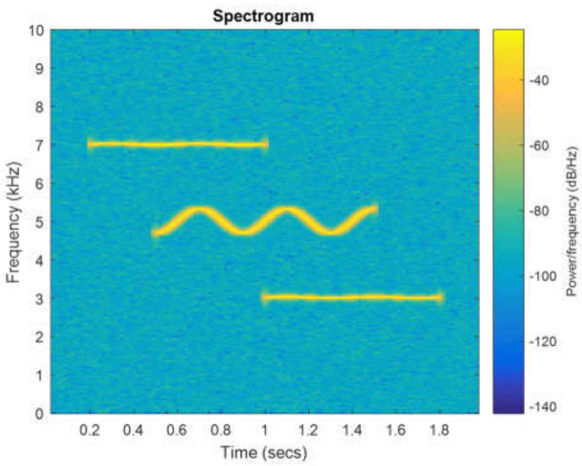
```
N=16; n= [0:N-1];
h=0.5*(1-cos(2*pi/N*(n+0.5)));
x=1+sin(n*2*pi/N).*cos(n*4*pi/N);
y=x.*h;
Y=fft(y);

H = zeros(1,N);
H(1)= ... ;
H( )= ... ;
...
X=fft(x);
Z=... ;
stem(abs(Y-Z)) % this error should be small < 1E-8
```

Exercise 3

Download from the Moodle platform the WAV file `audioSPEC.wav`. Then, execute the following Matlab commands to obtain the spectrogram of the signal contained in that file.

```
inpfile='audioSPEC.wav';
[x, FS]=audioread(inpfile);
N=1024; shift=N/4;
spectrogram(x,hann(N),N-shift,N,FS,'yaxis')
title('Spectrogram')
```



Based on this spectrogram, interpret the signal activity it reveals.