PDSI04MAY2021

Lecture #21

The auto-correlation and the cross-correlation
The Wiener Khintchine Theorem
The Parseval Theorem (particular case of W-
Computation of the A.C. and C.C.
using the DFT

$$X(z) \xleftrightarrow{\mathcal{Z}} x[n] \xleftrightarrow{\mathcal{F}} X(e^{j\omega})$$

$$X^*(z^*) \longleftrightarrow x^*[n] \longleftrightarrow X^*(e^{-j\omega})$$

$$X(z^{-1}) \longleftrightarrow x[-n] \longleftrightarrow X(e^{-j\omega})$$

$$X^*(1/z^*) \longleftrightarrow x^*[-n] \longleftrightarrow X^*(e^{j\omega})$$

auto-correlation function:

$$c_x[m] \triangleq x[m] * x^*[-m] = \sum_{k=-\infty}^{+\infty} x[k]\, x^*[k-m]$$

shift or "lag"

cross-correlation function:

$$c_{xy}[m] \triangleq x[m] * y^*[-m] = \sum_{k=-\infty}^{+\infty} x[k]\, y^*[k-m]$$

in the Fourier domain:

$$c_x[m] \xleftrightarrow{\mathcal{F}} C_x(e^{j\omega}) = X(e^{j\omega}) \cdot X^*(e^{j\omega})$$

$$= |X(e^{j\omega})|^2$$

$$C_X[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 e^{j\omega n} d\omega$$

Wiener - Khintchine Theorem

$$X(z)X^*(1/z^*) \overset{Z}{\longleftrightarrow} C_X[n] \overset{\mathcal{F}}{\longleftrightarrow} \underbrace{C_X(e^{j\omega})}_{\text{spectral density of energy}} = |X(e^{j\omega})|^2$$

$$C_X[0] = \sum_{k=-\infty}^{+\infty} x[k]\,x^*[k] = \sum_k |x[k]|^2 = E_X$$

$$C_X[0] = E_X = \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{|X(e^{j\omega})|^2}_{\text{spectral density of energy}} d\omega$$

Parseval Theorem

$$X(z)Y^*(1/z^*) \overset{Z}{\longleftrightarrow} C_{XY}[n] \overset{\mathcal{F}}{\longleftrightarrow} X(e^{j\omega})\,Y^*(e^{j\omega})$$

PROPERTIES

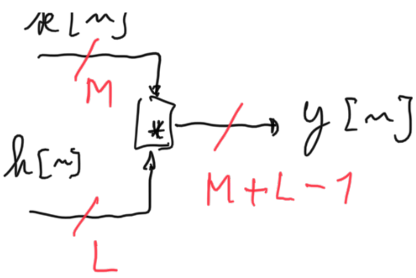
$$C_X[n] = C_X^*[-n] \qquad C_{XY}[n] = C_{YX}^*[-n]$$

$$|C_X[n]| \leq |C_X[0]| \qquad |C_{XY}[n]| \leq \sqrt{C_X[0]\,C_Y[0]}$$

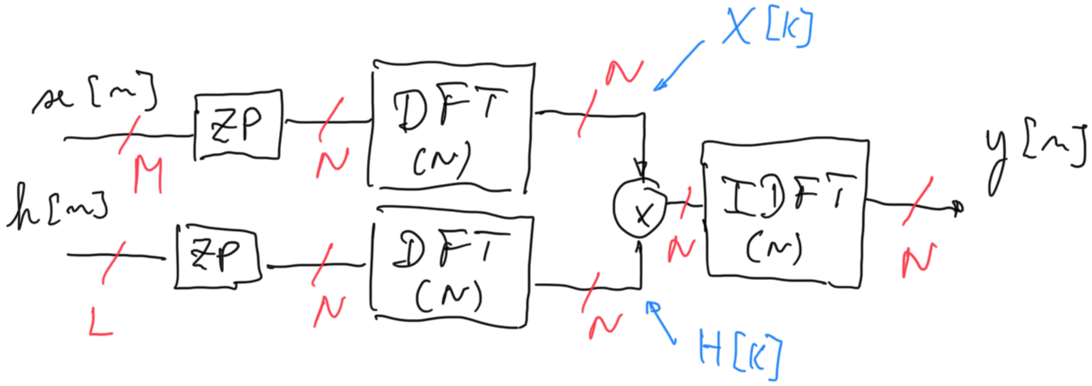
using the DFT

1. The "usual" convolution

$$y[n] = x[n] * h[n]$$



$$N \geq M + L - 1$$



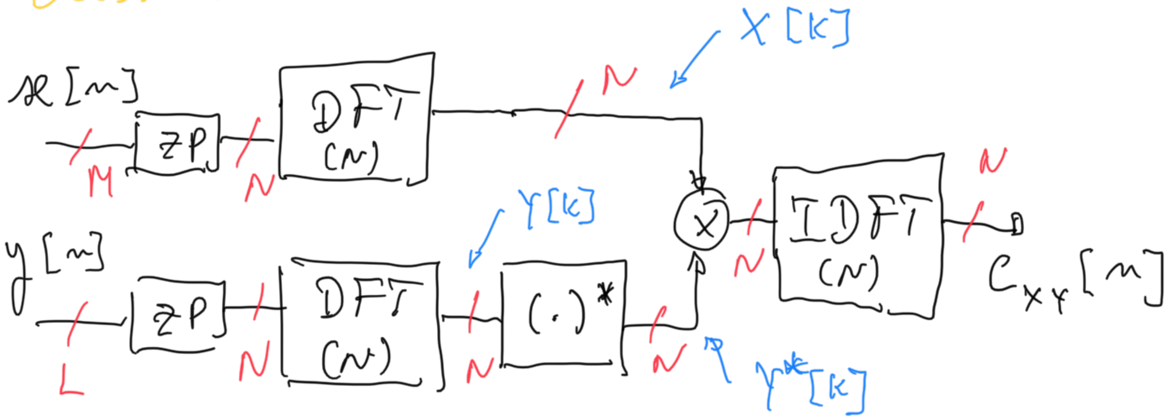
$$y[n] = x[n] \circledast h[n] \equiv x[n] * h[n]$$

after zero-padding

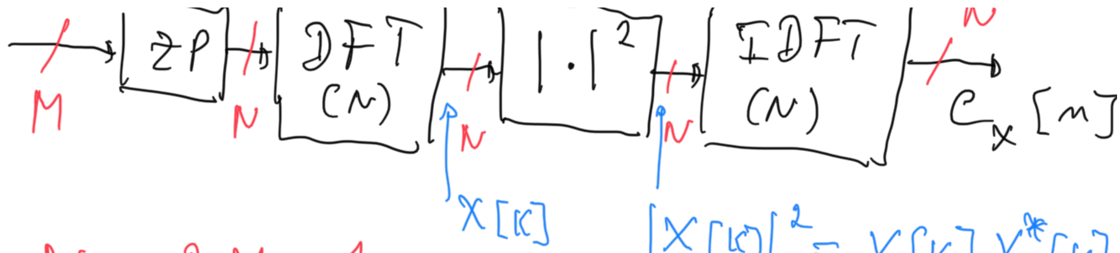
2. C.C. and A.C.

$$c_{xy}[n] = x[n] * y^*[-n]$$

cross-correlation



auto-correlation (particular case of cross-correlation)



$$|X[k]|^2 = X[k]\,X^*[k]$$

$$N \geq 2M - 1$$

ZP ≡ zero-padding